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ABSTRACT

This material is a product of a cooperative project of the Cincinnati Public Schools, the Detroit Public Schools, and CEMREL, Incorporated. The Teaching of Problem Solving (TOPS) project began in the 1980-81 school year under CEMREL's contract with the Basic Skills Improvement Office of the United States Department of Education. The approach used in the TOPS activities is to lead students through sequences of problem solving experiences presented in game-like and story situations. The material's design is based on the pedagogical theory fundamental to CEMREL's Comprehensive School Mathematics Program (CSMP), which emphasizes non-verbal languages to give pupils in grades K-6 access to mathematical ideas and methods necessary for posing interesting and challenging problems and solving them. It is noted that further information on the pedagogical and philosophical roots of TOPS is available through CSMP literature. The major portion of the document is divided into five sections: (1) Problem Solving in the String Game; (2) Problem Solving with the Minicomputer; (3) Problem Solving with the Hand Calculator; (4) Problem Solving through Detective Stories; and (5) Problem Solving with Arrows. Each section has a separate table of contents listing activities and games. Each item includes prerequisites, objectives, and instructional strategies. (Author/MP)

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ACTIVITIES FOR TOPS

A PROGRAM IN THE TEACHING OF PROBLEM SOLVING

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To Frédérique Papy,

who leads the way

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Reason is God's crowning gift ...
Sophocles, *Antigone*

FOREWORD

Problem solving receives considerable attention today. On the one hand, a decline of student mathematical achievement in the United States appears centered on students' problem-solving abilities, facilities with applications, and higher-level cognitive skills, rather than on their computational abilities.[1] On the other hand, the National Council of Teachers of Mathematics certified that "problem solving must be the focus of school mathematics in the 1980s", and further, "that the definition and language of problem solving in mathematics should be developed and expanded to include a broad range of strategies, processes, and models of presentation...."[2] But the paucity of problem solving in most school mathematics instruction is well documented.[3,4] Further, textbook series change only slowly from edition to edition, while textbooks already in schools change not at all. One remedy is to supplement standard programs through the use of well-chosen problem-solving activities. This is the object of the TOPS program.

The TOPS project began in the 1980-1981 school year under CEMREL's contract with the Basic Skills Improvement Office of the United States Department of Education. A cooperative project of the Cincinnati Public Schools, the Detroit Public Schools, and CEMREL, Inc., the project aims to provide cooperating teachers with good mathematical problem-solving activities with which to supplement their mathematics teaching. This book is a result of curriculum development efforts undertaken by project participants. Its activities received further trial and experimentation as the project continued.

TOPS problem-solving activities are based on the pedagogical theory fundamental to CEMREL's Comprehensive School Mathematics Program (CSMP), a complete elementary l vel mathematics curriculum for all students, kindergarten through grade six. In TOPS activities, as in the CSMP curriculum, students are led through sequences of problem-solving experiences presented in game-like and story situations. The curriculum uses non-verbal languages that give children at all levels access to the mathematical ideas and methods necessary for posing interesting and challenging problems and for solving them. Through these languages, the curriculum engages children immediately and naturally with the content of mathematics and its applications. Their development of mathematical sophistication does not depend on their linguistic development. In conjunction with the use of non-verbal languages, the spiral organization of the CSMP syllabus permits students to develop higher-level cognitive skills without waiting for mastery of computational technique. CSMP students work at problem solving throughout their school careers, for good habits of problem solving develop with time and experience.

Further information on the pedagogical and philosophical roots of the TOPS program is availiable in the literature of CSMP. The two programs are allied in their spirit and approach to mathematics education. Both are based on the belief that the development of reasoning abilities is closely related to the development of imagination, ingenuity, and intuition. Throughout, their activities are meant to promote open-ended thinking, searching among alternatives, testing of hypotheses, and other higher-level cognitive processes. They support teaching strategies that encourage creativity and allow freedom of exploration, while fostering sound intellectual habits. They recognize that success in such teaching requires curricular materials of high quality to serve as a vehicle for learning.

Inquiries and comments are welcome. Direct them to Joel Schneider, Director for TOPS, CEMREL, Inc., 3120 59th Street, Saint Louis, Missouri 63139.

Acknowledgments

Sixty-two teachers in Cincinnati and Detroit participated in the TOPS program. Their wholehearted effort in workshops, in testing materials, and in reporting their reactions and those of their students provided vital energy to fuel the program's growth and development.

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The program continued through refinement of the cooperating teachers' abilities to work with their students and colleagues and through development of the stock of activities. The participants in the collaboration are listed here.

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Materials

Several items are essential tools for the activities. Descriptions of them in the strand introduction and in the activities are sufficient to permit construction of models. However, standard models are available from CEMREL, Inc., as listed below. Copies of this book are also available.

Item	Item No.	List Price	School Price
Demonstration Minicomputer (4 boards with magnetic checkers)	8388902	17.20	13.76
Magnetic checkers	8388894	7.50	6.00
Individual Minicomputer sheets (60)	8388704	6.90	5.52
Sheets of checkers for Minicomputers (30)	8388712	5.90	4.72
A-Block String Game Kit	8388910	4.70	3.76
Numerical String Game Kit	8388852	4.70	3.76
Activities for TOPS	8381998	13.69	10.95

Prices are subject to change. All orders except those on institutional purchase orders must be prepaid. Please add 10% to cover shipping. Prepayment is required for all orders. Submit orders to CEMREL-TOPS, 3120 59th Street, St. Louis, MO 63139.

Using the Book

This book contains a variety of problem-solving activities that supplement a standard mathematics program at grades three through eight. The assumption throughout is that the introduction and major treatment of the various mathematical topics and concepts will be achieved through the standard curriculum. The activities here are organized into five strands, each with its own introduction, including advice on the use of materials. The activities are numbered within the strands. Each activity begins with a list of prerequisites and objectives.

Review the book by reading the introductions and some of the activities in each strand. Choose a strand that is attractive to you and that you think will be interesting and beneficial to your students. There are more activities provided than will be used by any one class during a year's course. One period a week devoted to the activities will allow your class to set a comfortable pace through the activities. Greater frequency may interfere with your basic curriculum. Lesser frequency may require additional time spent on review and reintroduction. Do activities from one strand so that students become familiar with the fundamentals of that strand. With that accomplished, you can move to another strand. When students are comfortable with several strands, choose among them as you think is appropriate. You will note that some of the activities include short exercises or games that can be used to fill the odd five or ten minutes. And some of the games are appropriate for small groups of students to play on their own initiative.

Your planning may be guided by a correlation between the activities and the scope and sequences of six commercial textbook series that we have developed and which appears on pages xiv-xv of this Foreword. Ten major content areas of the standard elementary mathematics curriculum are listed on the chart.

TOPS activities are identified by their numbers. To use the chart, find a textbook listing in one of the seven files. Note the grade level and unit or chapter in which the topic is covered in that text series (Counting Place Value, and Numeration, for example, is covered in grade 3: unit or chapter 2 of <u>Heath Mathematics</u>). Then consult the first rank to find the TOPS activities that include that topic. (Counting, Place Value and Numeration appear in Activities S8-13; W1, 2, 5-33; and so on.)

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In the ideal, you will have prepared to use this book through participation in a workshop introducing its tools and procedures. Prepare an activity by carefully reading the text, which is offered as a description, not as a prescription. For example, teacher's questions and students' answers in the activities are posed as guides to your experience and knowledge of your students. Be alert to matching the activities to your students' needs and abilities. The activities are useful in many situations and can be altered by substituting appropriate numbers in posing problems. Evidently a problem that is trivial to one student may be overwhelming to another. Rely on your experience and knowledge to make good choices which will engage your students' interest, which will challenge and excite their learning, and which will evoke their echoing Archimedes' "Eureka!"

References

[1] National Assessment of Educational Progress. Changes in Mathematics Achievement, 1973-1978. Denver: Education Commission of the States, 1979.

[2] "An Agenda for Action, Recommendations for School Mathematics for the 1980s". National Council of Teachers of Mathematics, April 1980.

[3] Lane County Mathematics Project. Introduction to the LCMP Mathematics Problem-Solving Programs, Eugene, Oregon: Lane Education Service District, 1979.

[4] Marilyn N. Suydam. Mathematics and 'The Urban Child'. Saint Louis: CEMREL, Inc., 1978.

A CORRELATION OF TOPS ACTIVITIES WITH SIX TEXTBOOK SERIES

Title of Text and Publisher	Counting, Place Value, and Numeration	Addition of Whole Numbers	Subtraction of Whole Numbers	Multiplication of Whole Numbers	Division of Whole Numbers
TOPS: A Program in the Teaching of Problem Solving Cemrel, Inc.	S8-13; W1, 2, 5-33; N3, 5-16; D1-11; H1-14; A1-5, 7-12, 14-16, 18-25; B1-6, 9, 10, 17	W1-8, 10-12, 14-18, 20, 22-33; N1-3, 6, 7, 9-11, 13-16; D1; H1-14; A1-9, 11-20, 22, 23, 25; B1-7, 13 15, 17	W9, 11, 17, 19, 22, 23, 25, 26, 28-33; N6, 9, 12-16; H1-6, 9-14; A1, 3, 4, 8, 10, 14-16, 19-21, 25; B1-3, 5, 7, 15, 17	S8-10, 12, 13; W1, 2, 7, 10, 21, 27; H4-6, 9, 10, 14; A1, 4-7, 9-14, 18, 20, 22, 25; B1-3, 6, 7, 9, 11-18	S9-13; H4-6, 9, 10, 14; A4, 13, 15-19, 21, 23, 24; B7, 12-18
Mathematics in Our World (1978 ed.) Addison-Wesley Publishing Company	3:12; 4:17, 21; 5:23; 6:29, 34; 7:1.1, 3.4; 8:7.1, 12.1, 12.2	3:11, 12; 4:17; 5:23; 6:29; 7:1.2; 8:7.3	3:11, 12; 4:17; 5:23; 6:29; 7:1.2; 8:7.3	3:13; 4:18, 19; 5:25; 6:30; 7:2.1; 8:8.1	3:14; 4:18, 20; 5:25, 26; 6:31; 7:3.1; 8:8.2
Heath Mathematics (1980 ed.) D.C. Heath and Company	3:2; 4:2; 5:1; 6:1; 7:1, 4; 8:1, 2	3:1, 3; 4:1, 3; 5:2; 6:1; 7:R, 1; 8:R	3:1, 4; 4:1, 3; 5:2; 6:1; 7:R, 1; 8:R	3:6, 10, 12; 4:5, 8, 11; 5:3; 6:2; 7:R, 1; 8:R	3:7, 10, 12; 4:5, 9, 11; 5:4; 6:3; 7:R, 1; 8:R
Holt Mathematics 1980: A Math For All Reasons, Holt, Rinehart and Winston	3:2; 4:2; 5:1; 6:1; 7:1, 3; 8:1	3:1, 3; 4:1, 3; 5:2; 6:2; 7:1; 8:2	3:1, 4; 4:1, 4; 5:3; 6:2; 7:1; 8:2	3:5, 8; 4:5, 7; 5:4; 6:3; 7:2; 8:2	3:6, 9; 4:5, 8; 5:5, 6; 6:4, 5; 7:2; 8:2
Mathematics, K-8 (1978 ed.) Houghton-Mifflin Company	3:2; 4:2; 5:2; 6:2; 7:3; 8:2, 14	3:1, 3, 10; 4:1, 3; 5:1; 6:1; 7:1; 8:1, 14	3:1, 4, 10; 4:1, 3; 5:1; 6:1; 7:1; 8:1, 14	3:5, 8, 9, 11; 4:4, 5, 7; 5:3, 5; 6:3; 7:2, 5; 8:1, 4, 14	3:6, 8, 9, 11; 4:4, 5, 8; 5:4, 5; 6:5; 7:2, 5; 8:1, 4, 14
The Laidlaw Mathematics Program (1978 ed.) Laidlaw Brothers	3:1, 2, 5; 4:1; 5:1, 9; 6:2; 7:1, 8; 8:1	3:1, 3, 7; 4:2, 8; 5:2; 6:2; 7:2; 8:3	3:1, 3, 8; 4:2, 8; 5:2; 6:2; 7:2; 8:3	3:5, 10; 4:4, 9; 5:4; 6:3; 7:3; 8:2	3:11; 4:5, 10; 5:5; 6:3; 7:3; 8:2
Scott, Foresman Mathematics K-8 (1980 ed.) Scott, Foresman and Company	3:1, 4, 10; 4:1; 5:1; 6:1, 5; 7:1, 6; 8:1, 7, 10	3:2, 5, 11; 4:2, 4; 5:1; 6:1; 7:1; 8:1	3:2, 6, 12; 4:3, 4; 5:2; 6:1; 7:1; 8:1	3:8, 9, 16; 4:7, 8, 9; 5:3; 6:2, 5; 7:2; 8:3	3:13, 14, 18; 4:11, 12, 17; 5:7, 8; 6:4, 5; 7:3; 8:1

Title of Text and Publisher	Decimals	Estimation	Geometry	Ratio, Proportion and Percent	Equations and Integers
TOPS: A Program in the Teaching of Problem Solving Cemrel, Inc.	D1-11; H4, 6, 9, 10; A18-20, 25; B11	W17, 20, 22, 25, 28, 30, 32; N14, 15; D5, 8, 11; H2-6, 9, 10	S1-7	H10, 14	S12; N1-16; H1, 4-6, 8, 10-13; A14-16, 25; B5, 7, 15, 16
Mathematics in Our World (1978 ed.) Addison-Wesley Publishing Company	3:16; 4:22; 5:24, 28; 6:29-31, 33; 7:1.2, 2.2, 3.2; 8:7.3, 8.1, 8.2	3:15; 4:17, 19; 5:24; 6:29, 30; 7:2.3; 8:7.4	3:11, 13; 4:17, 20; 5:23-25, 27; 6:30-33; 7:1.4, 4.5, 6.4; 8:7.5, 9.4, 11.4, 11.5, 12.5	4:21; 5:27; 6:33; 7:4.4, 5.1, 5.2; 8:9.3, 10.1, 10.2	3, 4: all chapters; 5:28; 6:34; 7, 8: all chapters
Heath Mathematics (1980 ed.) D.C. Heath and Company	3:3-5; 4:12; 5:8, 10, 12; 6:7, 8, 10; 7:2, 3; 8:1	3:3, 4; 4:3, 8, 9; 5:3, 8; 6:1, 2, 8; 7:2; 8:1	3:11; 4:10; 5:5; 6:4; 7:10; 8:7, 12	5:6, 10, 11; 6:10, 11; 7:8, 9; 8:5, 6, 8	3:10, 12; 4: all chapters; 5:12; 6:12; 7:4, 12; 8:9, 11
Holt Mathematics 1980: A Math For All Reasons, Holt, Rinehart and Winston	3:2; 4:10; 5:11; 6:6; 7:3, 4; 8:4, 5	3:3, 4; 4:3, 4, 7; 5:2-4; 6:2, 3, 5; 7:1, 2, 9; 8:2, 9	3:11; 4:11; 5:10; 6:10; 7:7, 8, 14; 8:10	6:11; 7:10; 8:9, 10	3, 4, 5: all chapters; 6:13; 7:9, 11, 12; 8:3, 9, 11, 12
Mathematics, K-8 (1978 ed.) Houghton-Mifflin Company	3:12; 4:9, 13; 5:8, 9; 6:4, 8, 12; 7:4, 6-8; 8:2, 3, 5	4:2; 5:11; 6:10; 7:3, 13; 8:2, 9	3:13; 4:12; 5:13; 6:11; 7:14; 8:10	5:12; 6:13; 7:6, 10, 12; 8:5, 8, 12	3, 4, 5, 6: all units; 7:15; 8:1, 13-15
The Laidlaw Mathematics Program (1978 ed.) Laidlaw Brothers	3:12; 4:11, 12; 5:9; 6:6; 7:6; 8:4	3:6; 4:2, 8; 5:2, 4; 6:2, 3; 7:1	3:4; 4:3; 5:3, 11; 6:5, 11; 7:5, 11, 12; 8:7, 8, 10, 11	5:9; 6:10; 7:10; 8:9	3:2, 5, 11; 4:6; 5:6; 6:1, 4, 14; 7:1, 4, 15; 8:3
Scott, Foresman Mathematics K-8 (1980 ed.) Scott, Foresman and Company	3:15; 4:5; 5:4, 10; 6:6, 7, 9; 7:4, 5; 8:2	5:1-3; 6:1, 2; 7:1, 2, 4, 5; 8:1, 2	3:3; 4:14; 5:13, 18; 6:16, 17; 7:13, 14; 8:13, 15	4:13; 5:9; 6:14, 15; 7:11, 12; 8:11, 12	3, 4, 5, 6, 7: all chapters; 8:4, 5, 18

Problem Solving In The String Game

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STRAND I: PROBLEM SOLVING IN THE STRING GAME

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INTRODUCTION

If the mathematics of sets has any place in school mathematics, it is precisely because it enables students to understand and to use the ideas of classification. The abilities to classify, to reason about classification, and to extract information from a classification are important skills for everyday life, for intellectual activity in general, and for the pursuit and understanding of mathematics in particular. In the ordinary course of events, one is not expected to deal with classification problems until one's command of the native language is well developed--not that an ability to classify requires special intelligence, but rather, it requires a means of expression.

Strings provide a precise, pictorial means of recording and communicating information about classification. Introduced into mathematical usage in the nineteenth century, and now customarily referred to as Venn diagrams, string pictures have had a long and useful existence in helping mathematicans and logicians to organize their information. Stripped of inappropriate formalism, the language of strings is useful for helping young students to think logically and creatively about sets and to report their thinking without extensive verbalizing.

Our use of strings is primarily in the context of the String Game, a logical game that provides a vigorous exercising of students' developing reasoning powers. The game is played with A-blocks (a set of geometric objects differentiated by size, shape, and color) and later with integers. By first using familiar objects with a few readily identified attributes, we introduce students to the procedures and logical intricacies of the game. As students' experience grows, the game is enhanced by more complex versions of the String Game with A-blocks or by introducing the richer mathematical context of integers and their attributes.

By way of example, a brief description of one version of the game and its preparation is included here.

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Materials

Prepare a chart showing all of the attributes to be used in the game and a card for each attribute. Prepare game pieces, one for each of the numbers that are listed below. Put magnetic material or loops of masking tape on the backs of the pieces and the fronts of the string cards so that they will adhere to the board. An alternative to masking tape is a plastic caulking compound that will stick, but not dry out.

GAME PIECES

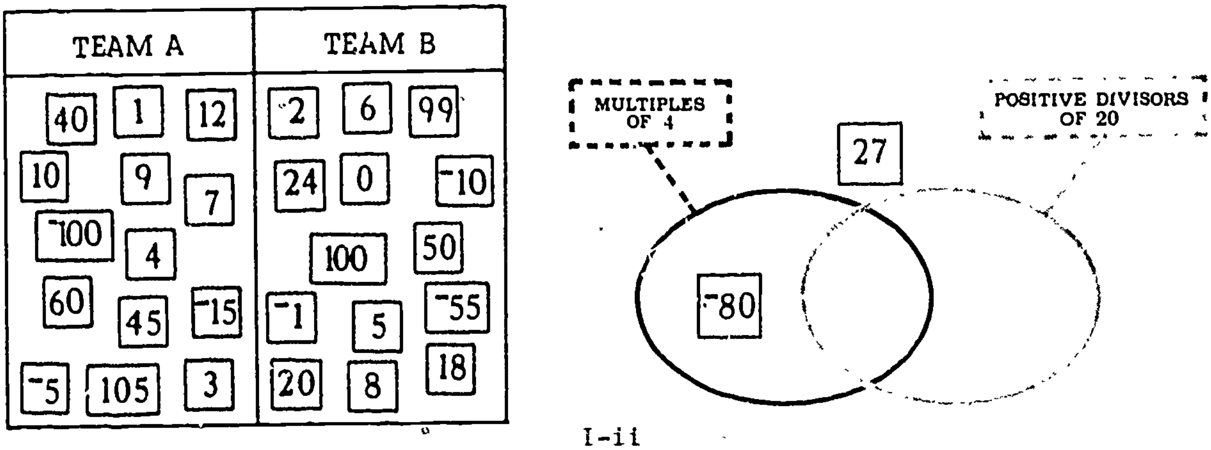
-100	-80	-55	-15	-10	-5
-1	0	1	2	3	4
5	6	7	8	9	10
12	18	20	24	27	40
45	50	60	99	100	105

ATTRIBUTES

MULTIPLES OF 2	MULTIPLES OF 3	MULTIPLES OF 4	MULTIPLES OF 5
MULTIPLES OF 10	POSITIVE DIVISORS OF 12	POSITIVE DIVISORS OF 18	POSITIVE DIVISORS OF 20
POSITIVE DIVISORS OF 24	POSITIVE DIVISORS OF 27	LARGER THAN 50	LARGER THAN -10
SMALLER THAN 50	SMALLER THAN -10	ODD NUMBERS	POSITIVE PRIME NUMBERS

Preparation

Draw two (or three) large, overlapping strings on the board, using two (or three) different colors. Next to each string attach a string card face-down. Place two of the game pieces correctly in the string picture to start the game. Divide the rest of the game pieces evenly into two sets, one for each of the teams. The illustration below shows a sample set-up for a game; dotted boxes indicate face-down labels.



Object of the Game

Each team tries to place all of its game pieces correctly in the string picture according to the face-down string cards. The winning team is the one that first identifies the face-down cards correctly according to the rules.

Rules of the Game

1. The game is played in silence. Each student should have the opportunity to analyze the game. Infringement of this rule by anyone is penalized by the talker's team losing its next turn.

2. The teams alternate, and all of the players on each team rotate turns. A player comes to the board and selects a piece from the team's collection to place in one of the regions of the string picture.

3. You are the judge. If the piece is correctly placed, say "yes". The piece remains in the string picture and the player immediately has a second turn (no player may have more than two consecutive turns). If the piece is incorrectly placed, say "no". The player returns the piece to the team's unplayed collection and play passes to the other team.

   As an aid in judging, prepare a crib-sheet showing the correct position of each game piece, or at least reminding you of what is on the face-down attribute cards. If at any time you discover that you have made an error, say so immediately and rectify the mistake either by moving an incorrectly placed piece to its correct region or by replacing in the string picture a correctly placed piece that had been rejected.

4. When a team has correctly placed all of its pieces, the player who placed the last piece may immediately attempt to identify each of the string cards. If these are all correct, the team has won. If any mistake is made (even if one of the identifications was correct), simply indicate that the identification is incorrect and let the game continue.

Rules of the Game (continued)

5. If a team has exhausted its stock of game pieces but fails to identify the attribute cards correctly, it continues on its turn to attempt to identify the strings, while the other team works to place its remaining game pieces.

The game may be used at several levels. At the very simplest and introductory level--in the String Game with numbers, for example--students may just practice placing numbers in the string picture according to open labels, thereby practicing the related arithmetic facts. In this way they develop their sense of the relations among the various attributes and the idea of the game itself. Reducing the attribute list by deleting "divisors of 12", "divisors of 18", "divisors of 20", "divisors of 24", and "divisors of 27" provides a game with more emphasis on multiples.

While the arithmetic emphasis of the game is on the mental arithmetic of multiplying, dividing and ordering, students also develop their logical faculties in reasoning about the possibilities for the hidden attributes from limited information given in the string pictures. When students have a good command of the arithmetic involved, reasoning itself becomes the more important element of the game.

Appearing throughout the CSMP curriculum, the String Game has received intensive study and development. In a heterogeneous class, it is singularly powerful in involving students of all levels because each student can contribute to the progress of the game. Many times in each game, there are plays discernible by very weak students at the same time that plays exist that require complex reasoning. In any event, the game admits relatively few moves that are wrong or bad, for even the knowledge that a number does not belong in a certain region conveys information about the situation. The same features make the game very useful for groups of students of low achievement. In practice, students respond to the String Game with excitement and play it with enthusiasm.

ACTIVITY S1: PREPARATION FOR THE STRING GAME

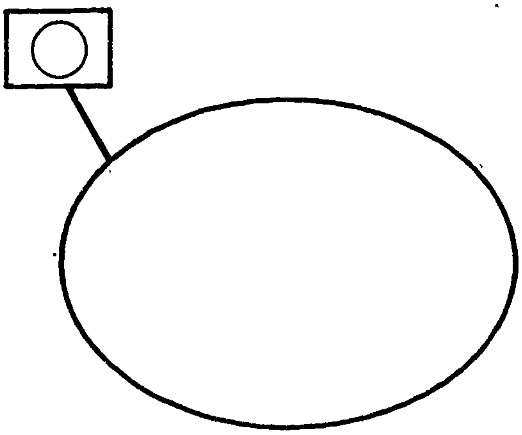
PREREQUISITE: None

OBJECTIVE: Students will identify attributes of the A-Blocks and place pieces correctly in string pictures.

Set up the team board. Put the twenty-four A-Blocks into a box about the size of a shoe box. Sort them so that each A-Block can be located quickly.

Have students guess the various shapes and colors represented among the collection of pieces in the box. Mention that there are two sizes--big and little. When the shapes--square, circle, and triangle--and the colors--red, yellow, blue, and green--have been identified, have students name pieces from the box by describing three attributes--size, color, and shape for each. Thus a student might say, "large blue triangle," but "large triangle" or "red square" are not sufficiently precise. As the pieces are named, place them on the board in one or the other team's space. If a piece is named that is already on the board, simply point to it and ask for another. Encourage students to be thinking about which pieces they will ask for when called upon.

On the chalkboard draw a large red string and label it ◯ .

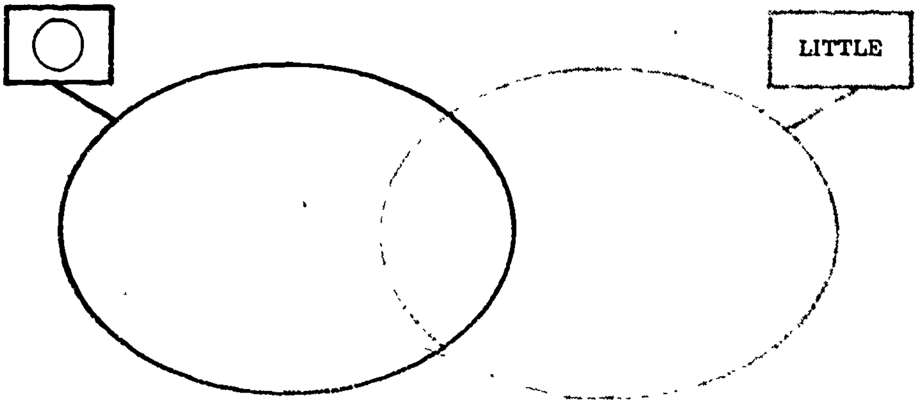


T: All the pieces that are circles belong inside this string. All other pieces belong outside it.

Take several pieces, one at a time, off the team board. Ask whether each belongs inside or outside the red string and let the students decide its correct placement. When the class is clear about the placement of the pieces, return all of them to the team board.

Draw a large blue string overlapping the red string and label it LITTLE.

Note: Throughout these activities we will refer to the left string as "red" and the right string as "blue". If there is a third string, it will be referred to as "green".



T: All pieces that are little belong in the blue string. All other pieces belong outside it.

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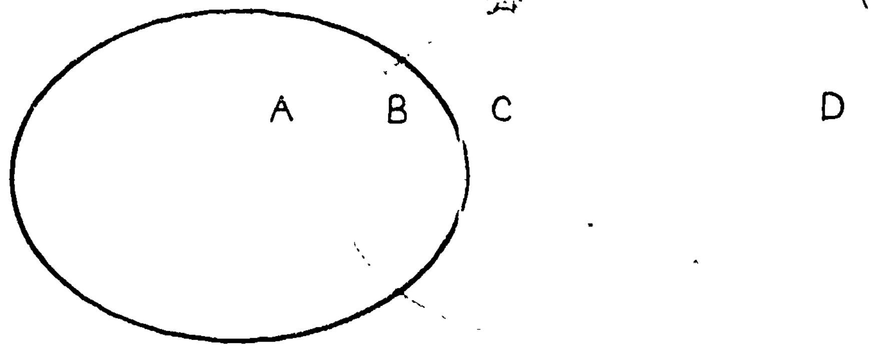
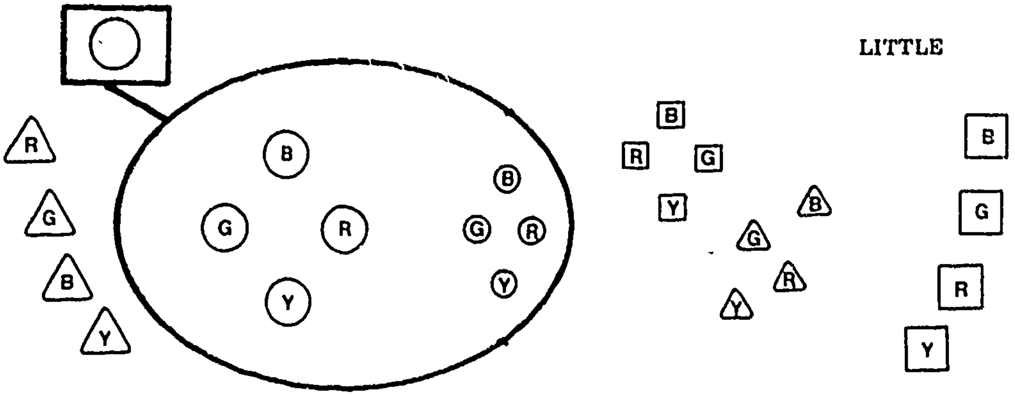
Note: Some students may need to be reminded of the region outside both strings. To aid them, and to permit easy reference in discussing the placement of game pieces, label the four regions "A", "B", "C", and "D".



T: The teams will take turns placing the pieces from their side of the game board correctly in the string picture. If the piece is placed correctly, I will say "yes" and the piece remains in the string picture. If it is placed incorrectly, I will say "no" and the piece must be returned to your team's side of the board. The first team to place correctly all of its pieces in the string picture wins.

Teams alternate and members of a team take turns while playing the game. This illustration shows the correct placement of the pieces.



ACTIVITY S2:   THE STRING GAME WITH A-BLOCKS

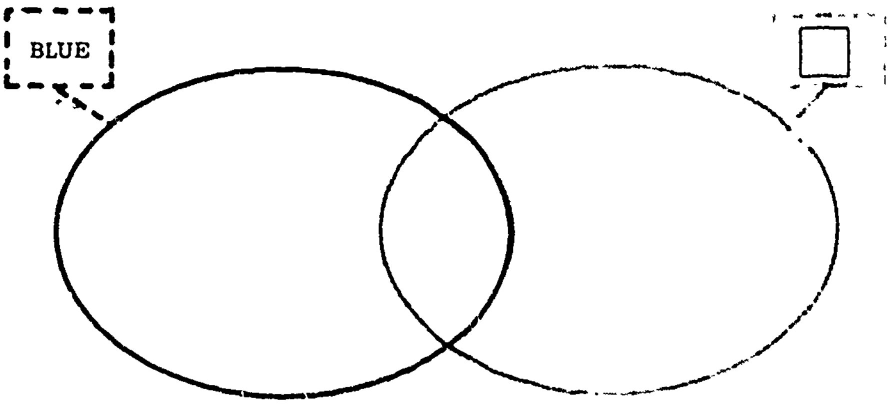
PREREQUISITE: Activity S1

OBJECTIVE: Students will play the String Game to determine hidden labels.

Conduct a short warm-up activity like the last exercise of Activity S1.

Distribute half the A-Block pieces to each side of the team board. Place the string card BLUE face-down near the red string and place the string card face-down near the blue string. Place two A-Block pieces correctly in the picture as starting clues. Each team should have eleven pieces on the team board.

Note: Hidden labels will be indicated by dotted boxes.



T: We'll play the game with the string labels hidden. Teams will take turns trying to place their pieces. We'll play with a bonus rule: players who place their first piece correctly get a second turn.

When a team has correctly placed all of its pieces, the player who placed the last piece can tell what the strings are for. If the player identifies both labels correctly, that team wins. If either label is incorrect, it will be the other team's turn.

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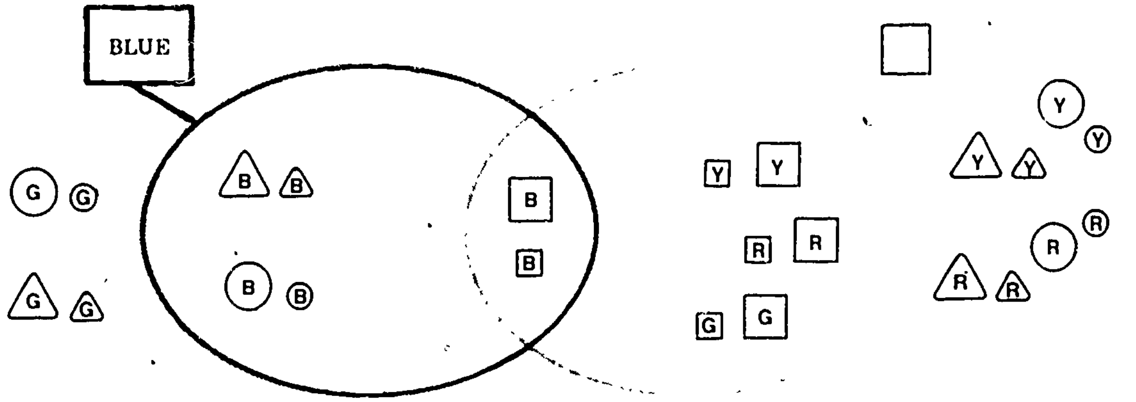
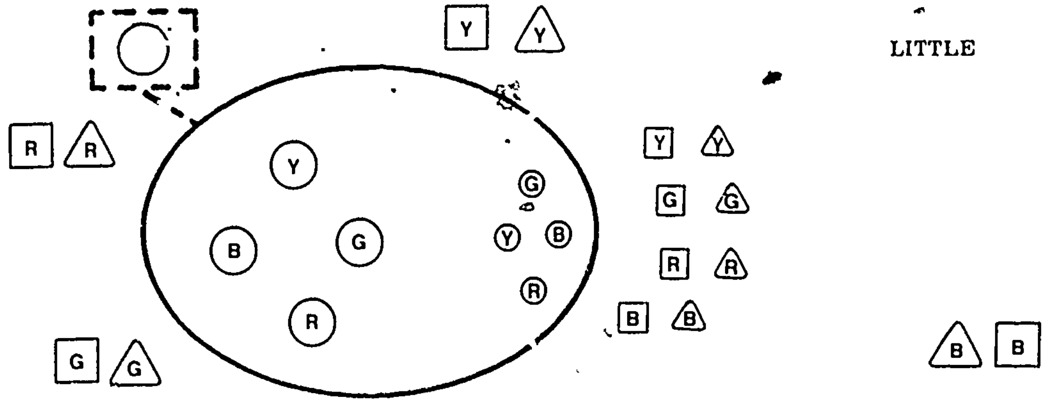
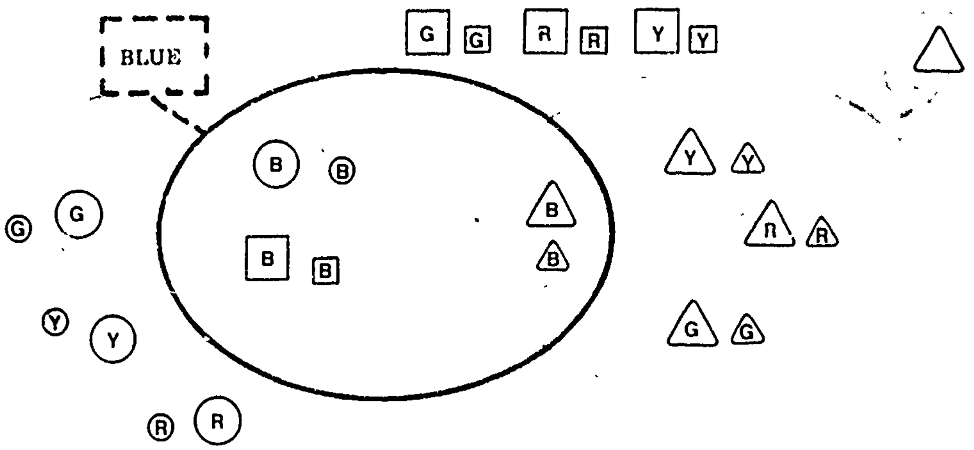
ERIC

Play the game. The next illustration shows correct placement of the pieces.



Play as many games as time and interest permit. Play any games omitted now at a later time.





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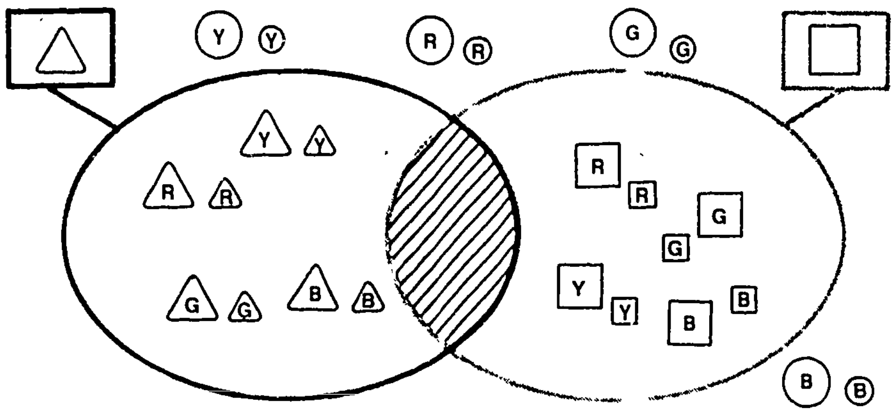
ACTIVITY S3: INTRODUCTION TO EMPTY REGIONS

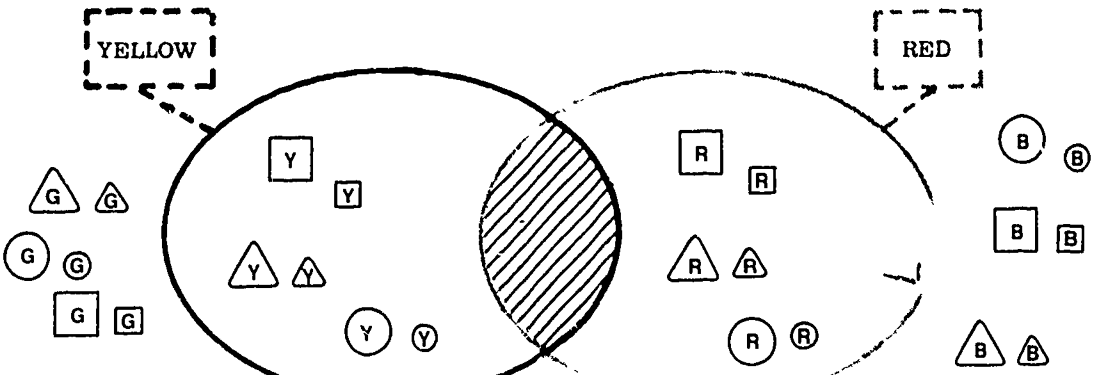
PREREQUISITE: Activity S2

OBJECTIVE: Students will recognize an empty region as a region with no pieces because of contradictory demands of the labels.

Play a String Game similar to S1, with the string cards and showing, as indicated in the next illustration. When the game is complete, discuss the empty region. No piece can be correctly placed in the center region because no piece is both a triangle and a square. Hatch the region as shown and explain that hatch marks indicate that no more pieces belong in that region.



Here is another game with an empty region. Play with labels hidden.

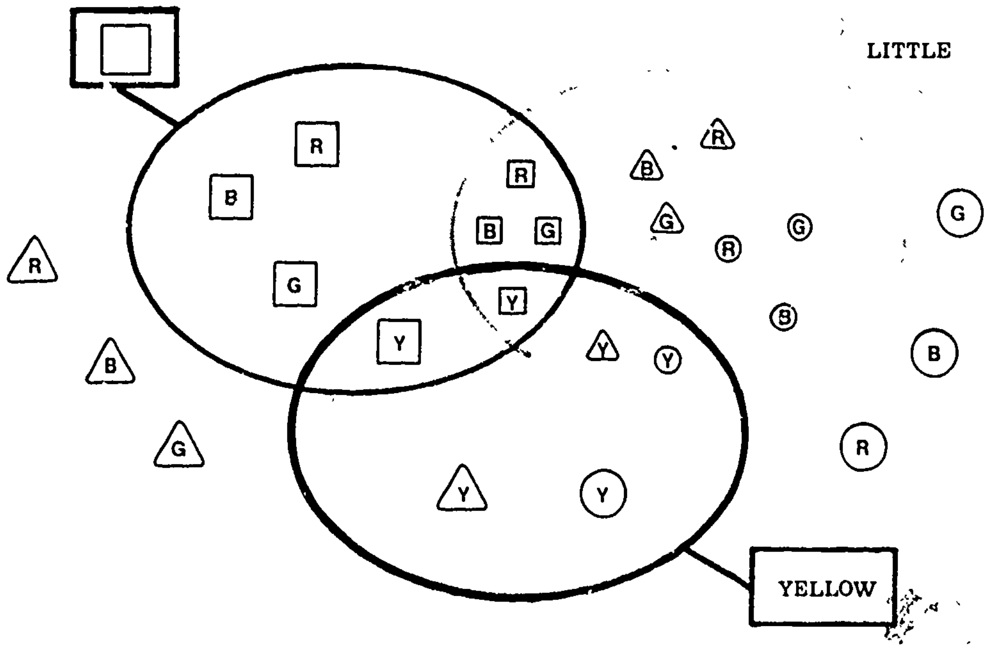


ACTIVITY S4: THE STRING GAME WITH THREE ATTRIBUTES

PREREQUISITE: Activity S2

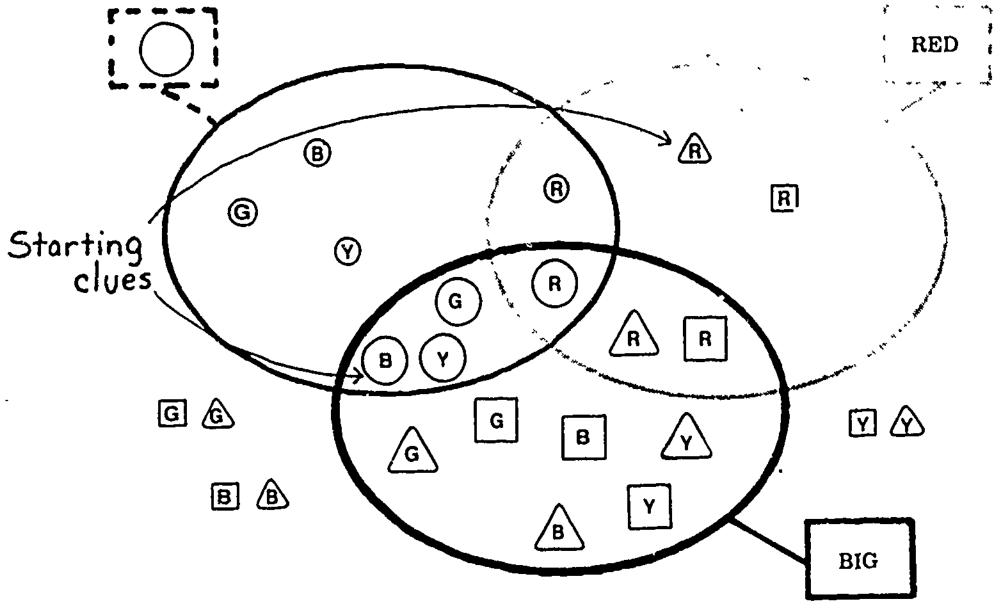
OBJECTIVE: Students will play the String Game using three attributes.

Display the A-Blocks and draw a string picture on the board with three strings as shown below. Do not place any A-Blocks in the string picture before beginning the lesson. Trace each region and name its label. Then point to a region and ask, "Which pieces belong here?" Discuss the correct placement of the pieces suggested. Be sure to include the region outside all three strings. Continue this activity with three or four other regions. Pick several pieces one at time that are not already in the picture and ask, "Where does this piece belong?" None of the eight regions of this picture are empty. Like a two-string picture, each region may be labeled for ease in discussing the picture and the placement of pieces.

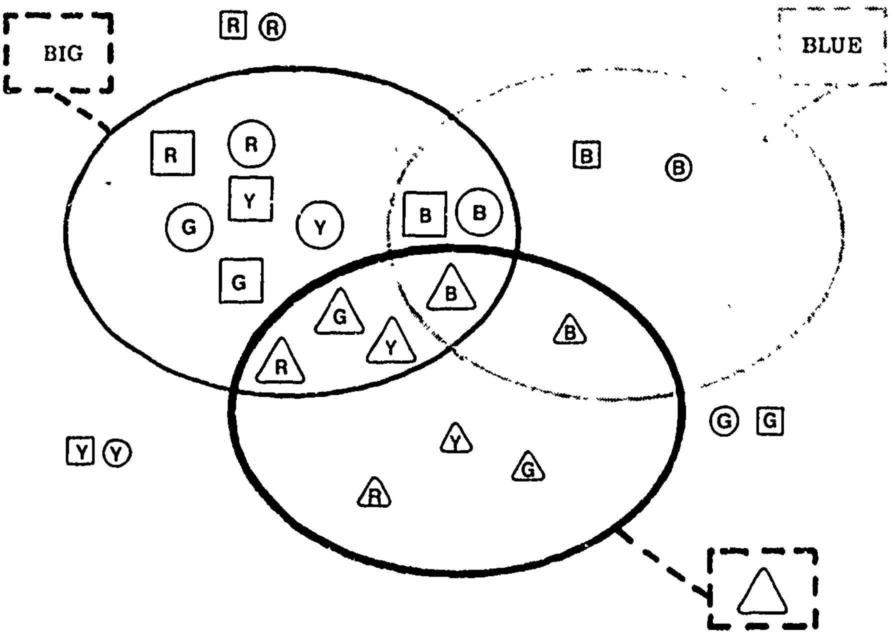


23

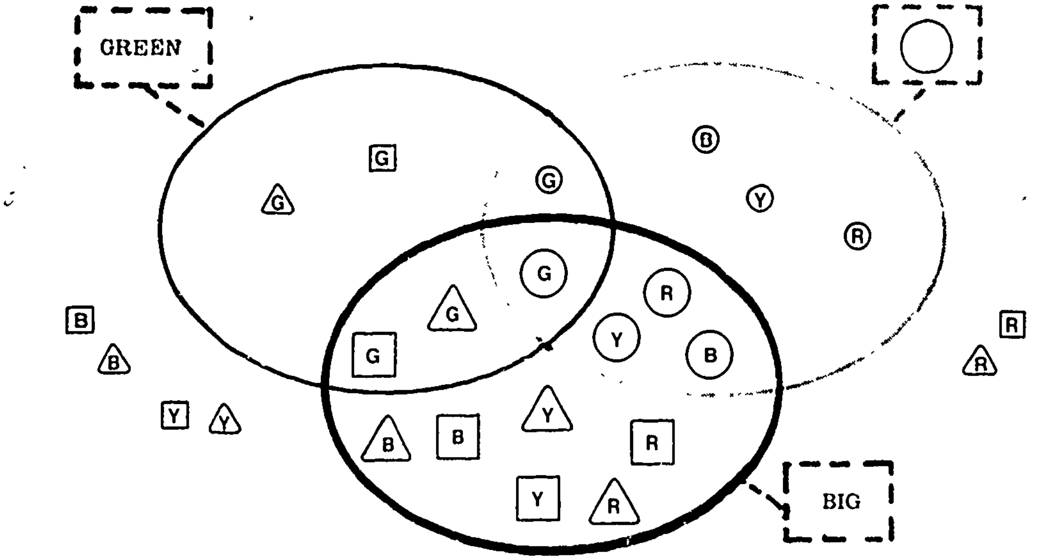
Next, play a game with two labels hidden and one showing. Give the two starting clues shown here.

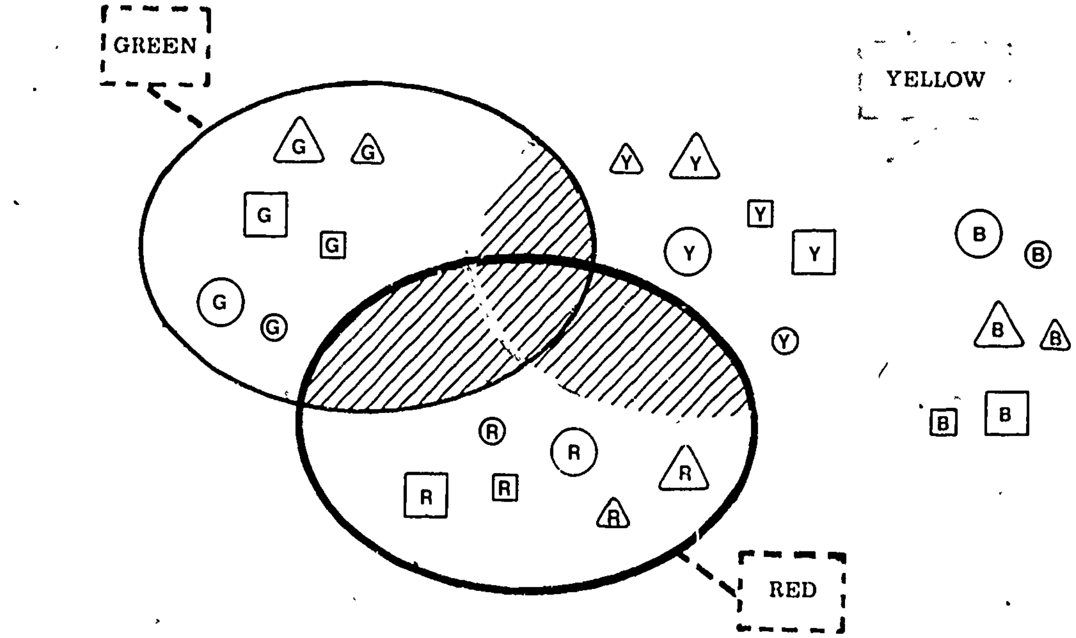


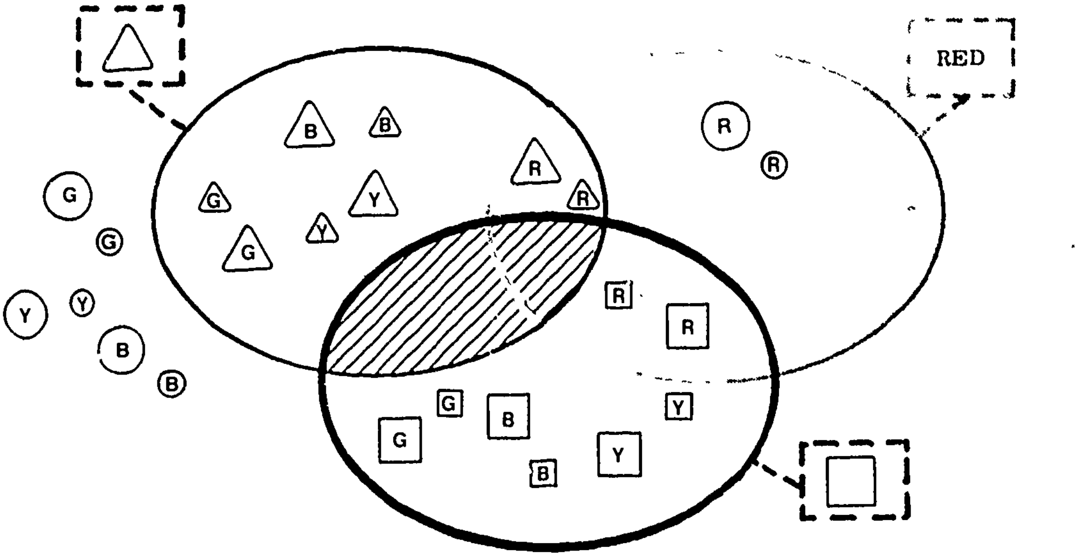
Now play a game with all labels face-down. Four of the many possible games are shown here.



3υ


GREEN
BIG


GREEN
YELLOW
RED

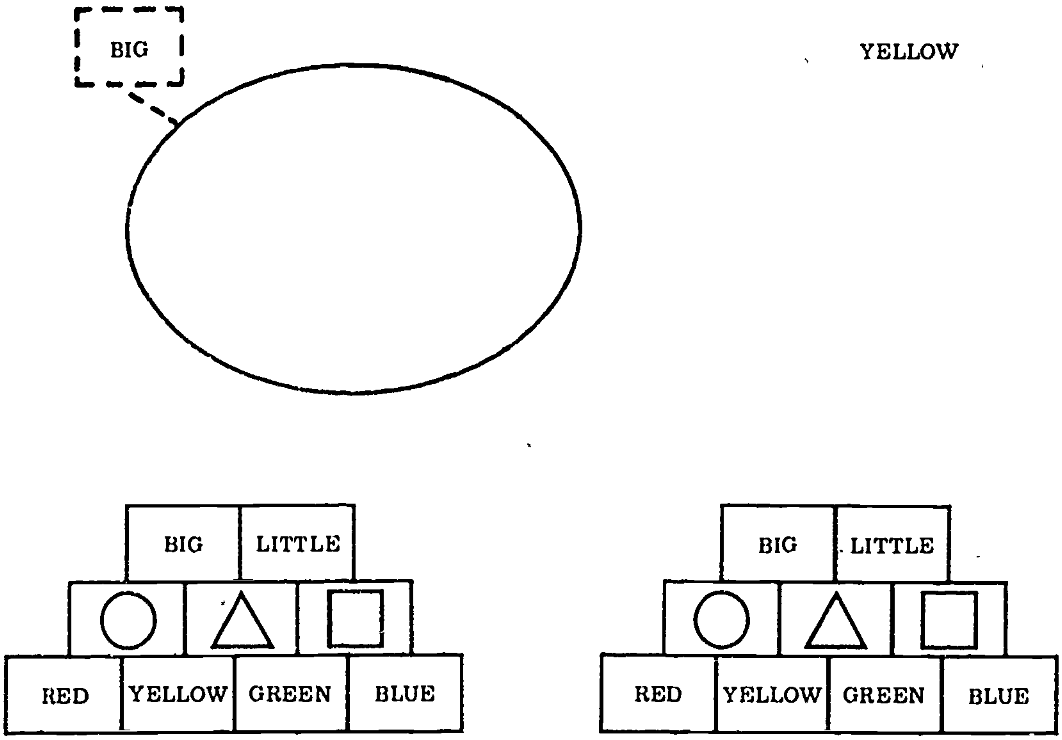

RED

ACTIVITY S5: ANALYSIS OF THE STRING GAME WITH A-BLOCKS

PREREQUISITE: Activity S3

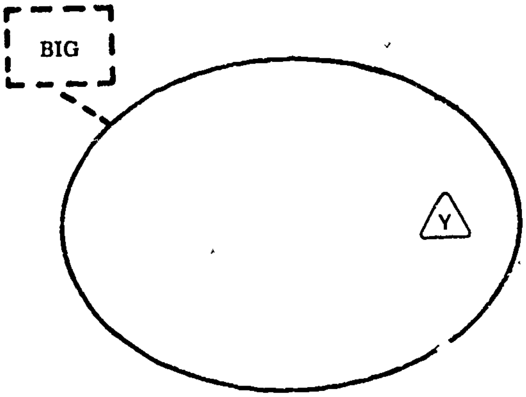
OBJECTIVE: Students will analyze the placement of pieces in a String Game to determine the string labels.

Prepare to play the String Game as illustrated below. Distribute the game pieces and tape two A-Block String Game Posters to the board, one for the red string and one for the blue string.



T: We will play the String Game today, but first we will look at the information we get when a game piece is correctly placed in the string picture. Determining the labels will be like solving a detective story; each piece correctly placed is a clue.

YELLOW



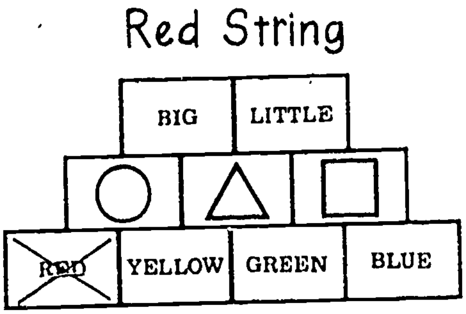
T: The first clue is that the big yellow triangle belongs inside both strings. What information does this give about the strings? Are there any labels (point to one of the posters) that either string cannot have?

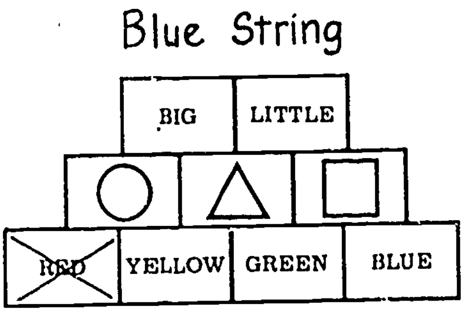
S: The labels cannot be RED because a yellow piece is inside both strings.

T: So we can cross out RED on which chart?

S: We can cross out RED on both charts since the yellow triangle is inside both strings.

Use a felt-tip marker or crayon to cross out RED on both charts.





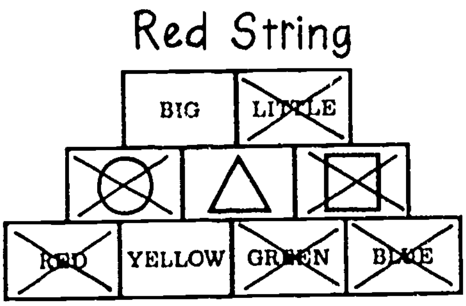
In the same manner, let students continue analyzing the situation. Each time they correctly suggest crossing out a label on one chart, they should notice that the same label can be crossed out on the other chart. A piece in the center region gives the same information about both strings.

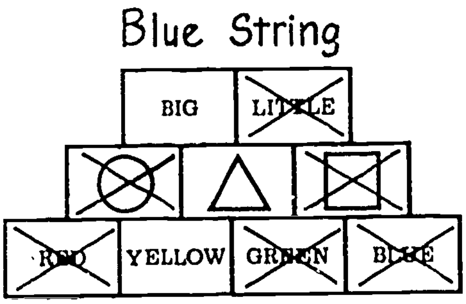
A student may suggest incorrectly that some label be crossed out on the charts and you will need to point out the error.

S: Cross out BIG on the chart for the blue string.

T: But this piece (point to the yellow triangle) is big. If the label were BIG, this triangle would be correctly placed.

If your class exhausts this clue's information, only three possibilities will remain for each string. Do not insist that the students find all the attributes that can be crossed out if no more suggestions are forthcoming.

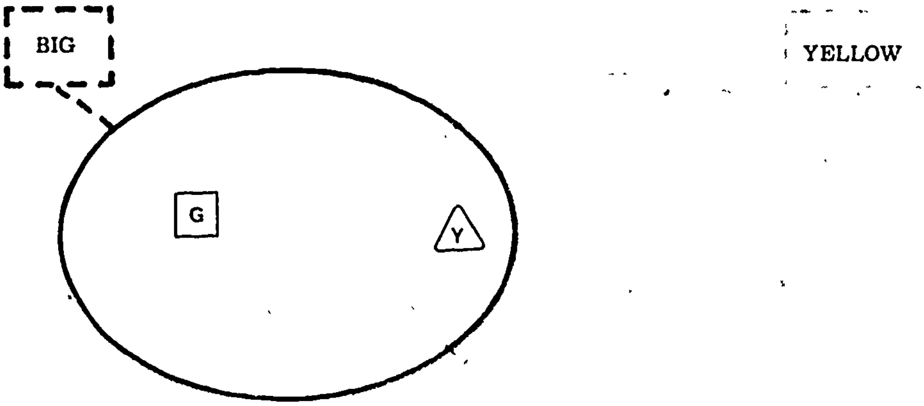




34

Give another clue.

T: The big green square belongs in the red string, but not in the blue string.



Ask the class if there are any other labels that the red string cannot have.

T: Could the label on the red string be △ ? (Trace the red string as you say this.)

S: No, because the green square is not a triangle.

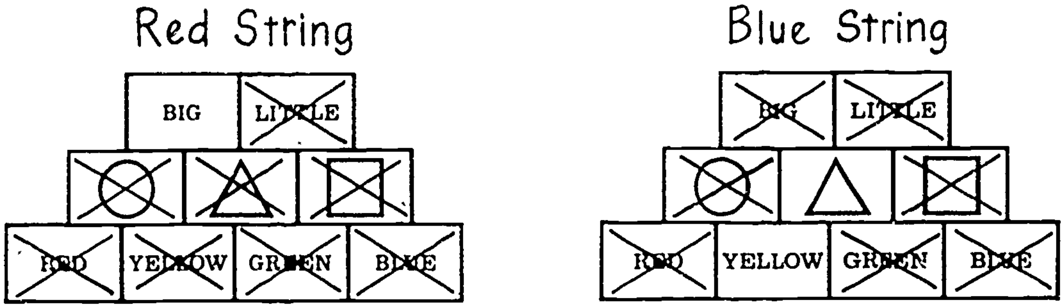
T: Could the label on the red string be BIG?

S: Yes, because both shapes are big.

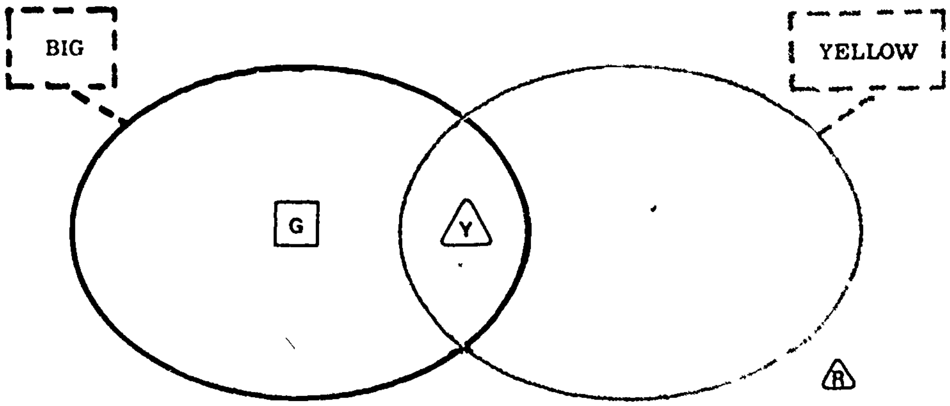
Do not cross out BIG on the red chart. Continue crossing off labels until you determine that the red string label must be BIG.

ERIC

Consider the remaining possibilities for the blue string. The analysis involved is slightly different for this string because the large green square is outside the blue string. BIG is eliminated from the chart for the blue string because it would require the large green square to be inside the blue string. Your charts should look like the following at the end of the discussion.



T: Another clue is that the small red triangle belongs outside both strings.

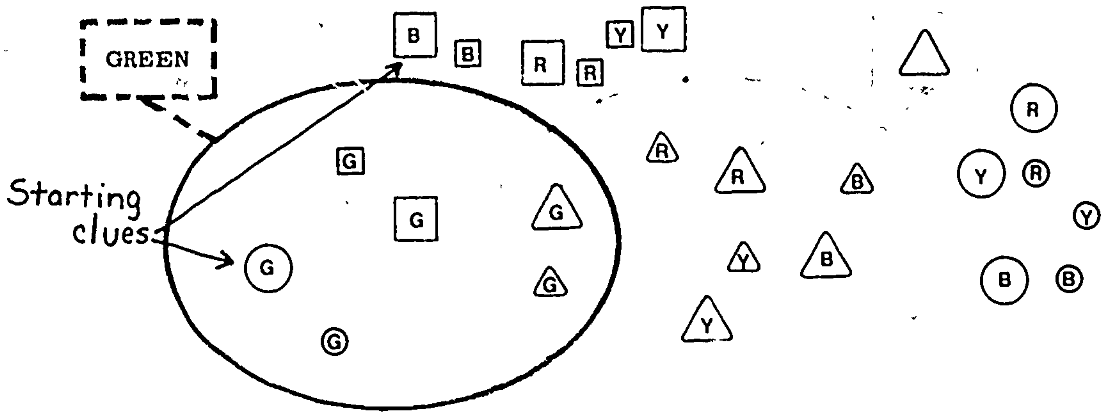


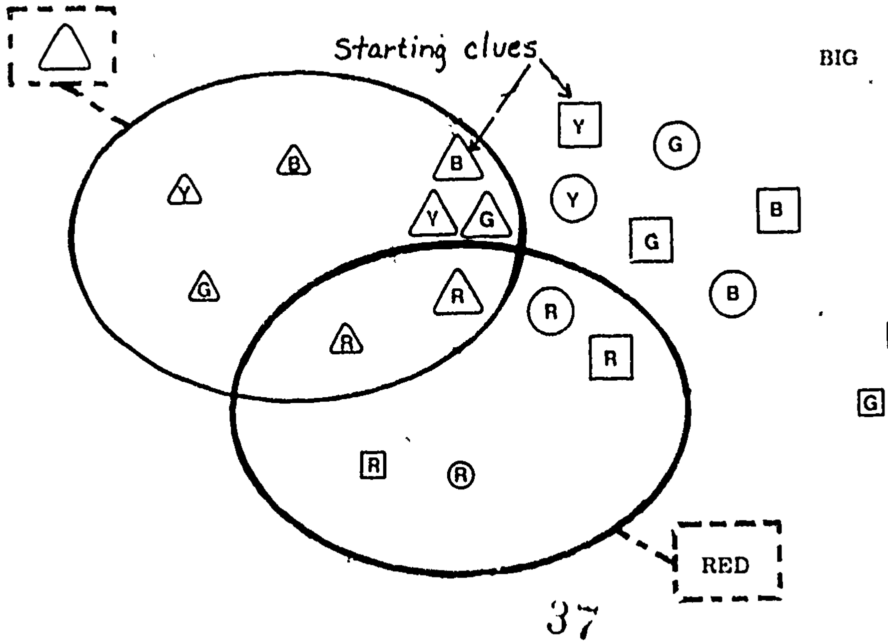
From this clue, your class should conclude that the blue string's label is YELLOW. The red triangle is outside the blue string, so △ can be crossed out on the chart for the the blue string.

Reproduce and distribute String Game analysis sheets found on page I-40.

T: We will play a String Game now. Each of you has a chart of the possible labels for the red string and the blue string. Try to use this chart during the game to help you discover what is on the hidden labels. Try to cross out some of the possibilities as pieces are put in the picture.

In order to give the students time to do some of their own analysis between plays, pause briefly after placing the starting clues and between turns if a player has placed a piece correctly. Such pauses in the game encourage students to use the clues in crossing out labels on their individual charts. Two of the many possible games are shown below.





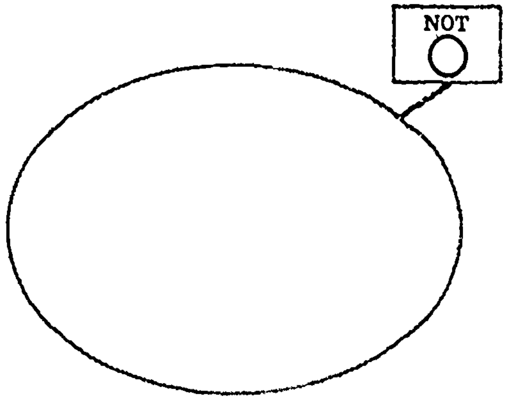
ACTIVITY S6: INTRODUCTION TO NOT-CARDS

PREREQUISITE: Activity S2

OBJECTIVE: Students will play the String Game with A-Blocks using not-cards.

Quickly review the attributes of the A-Blocks--that is, their sizes, colors, and shapes.

T: There are some other possibilites for string labels. Show the class the string card NOT ◯ . (Read, "not circles".) All pieces that are not circles belong in this blue string.



Ask students to identify pieces that belong inside the string and then some pieces that belong outside the string. All circles belong outside the string and all other pieces belong inside the string.

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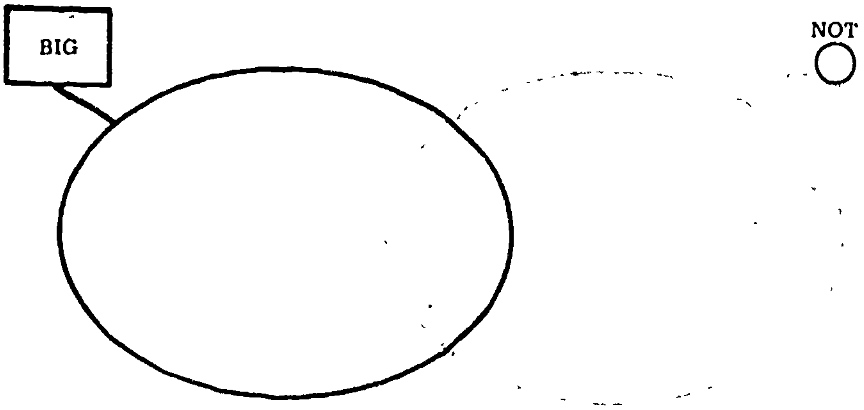
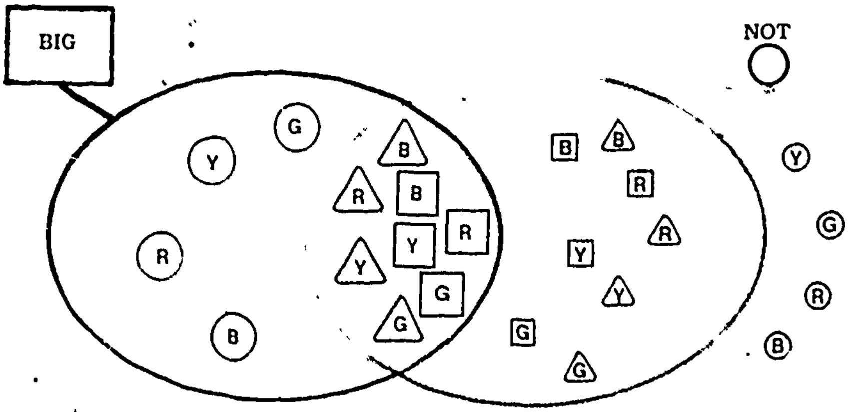
Clear the picture of all game pieces and draw a red string overlapping the blue string. Label the red string BIG.



Call on students to place game pieces in the picture. Discuss with the class why each suggested placement is correct or incorrect. At least one piece should be placed in each region. The following picture shows the correct placement of the twenty-four game pieces.



Clear the picture of all game pieces and remove the string labels. Ask students to name other new string labels. Display each string label as it is mentioned. If a student suggests NOT LITTLE or NOT BIG, discuss why such string cards are not needed. Pieces that are not little are the same as those that are big, and pieces that are not big are the same as those that are little.

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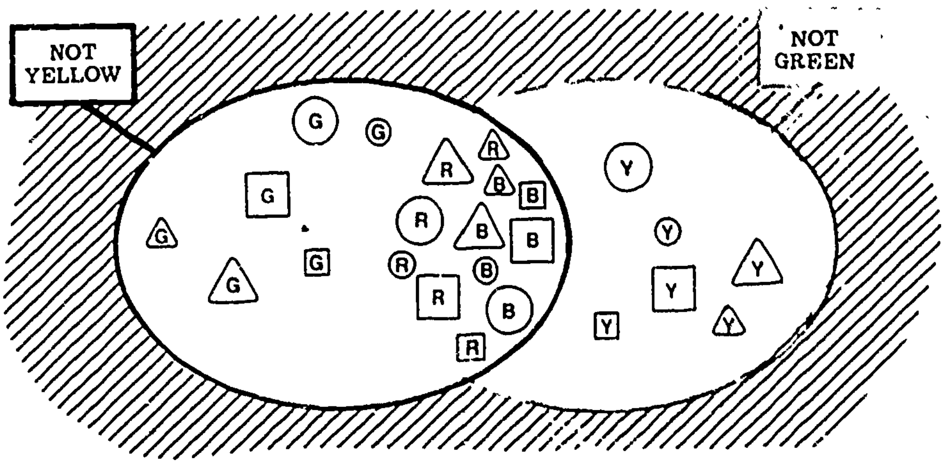
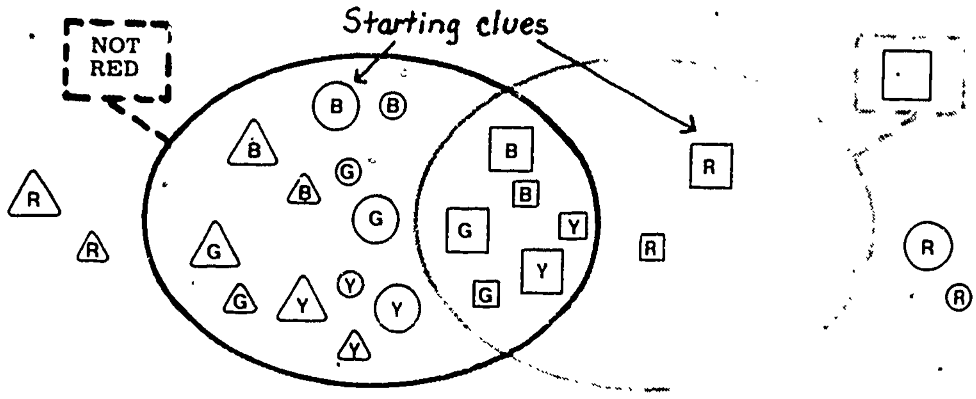
Tape a copy of the A-Block String Game Poster, which shows all sixteen attributes, to the board. Draw or display the team board.

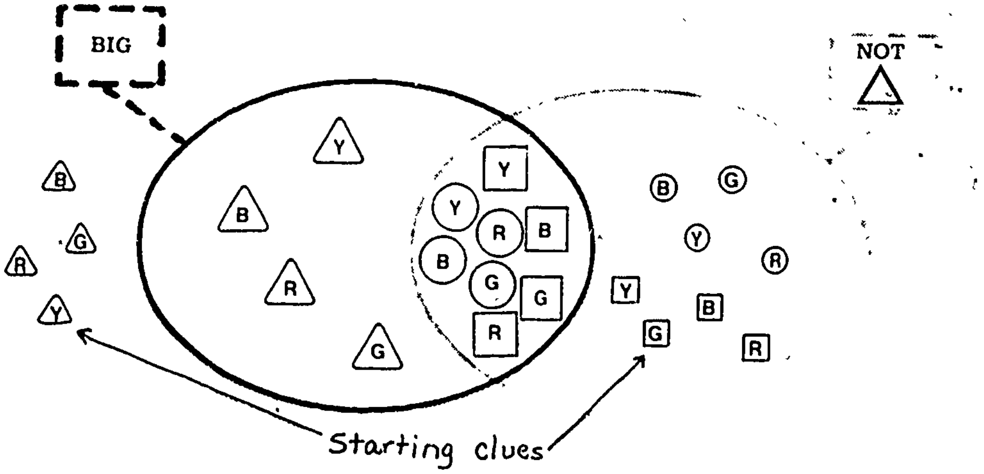
RED	YELLOW	GREEN	BLUE
NOT RED	NOT YELLOW	NOT GREEN	NOT BLUE
○	△	□	BIG
NOT ○	NOT △	NOT □	LITTLE

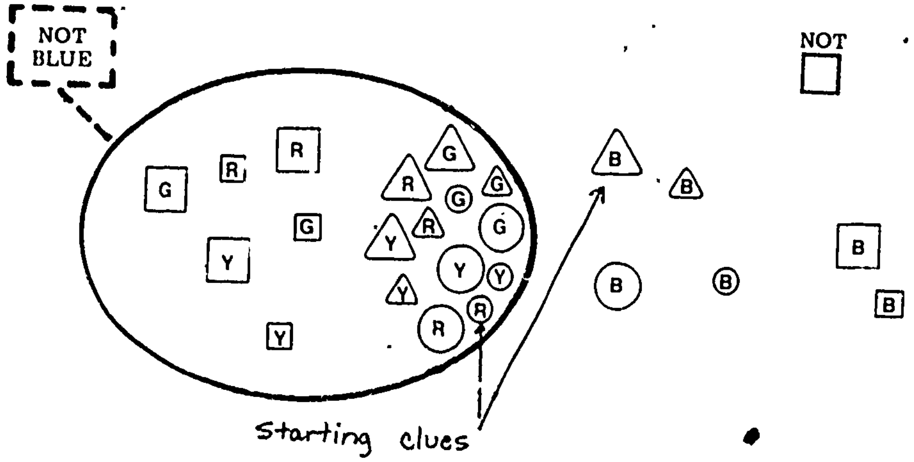
Divide the class into two teams, Team A and Team B. Distribute the game pieces on the team board. Label the strings as shown below. Since the labels are showing, no starting clues are necessary. Continue as in the final exercise of Activity S1, page I-1. The following illustration shows correct placement of the twenty-four game pieces.



Repeat the exercise with another choice of labels, if you wish. When the class is comfortable with the not-cards, play the String Game with hidden label cards as described in Activity S2, but this time include the not-cards. Each time you play the game, remind the class that there are sixteen possibilities. Two of the many possible games are shown on the next page with starting clues indicated.


Starting clues
NOT
RED


BIG
NOT
Starting clues

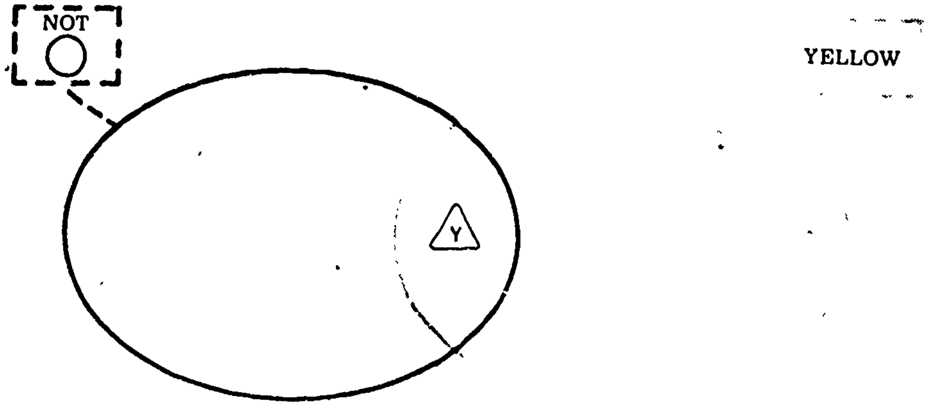

NOT
BLUE
NOT
Starting clues

ACTIVITY S7: ANALYSIS OF THE STRING GAME WITH NOT-CARDS

PREREQUISITE: Activities S5 and S6

OBJECTIVE: Students will analyze the placement of pieces in a String Game using not-cards to determine the string labels.

Prepare to play the String Game as illustrated below. Tape two A-Block String Game Posters on the board, one for the red string and one for the blue string.

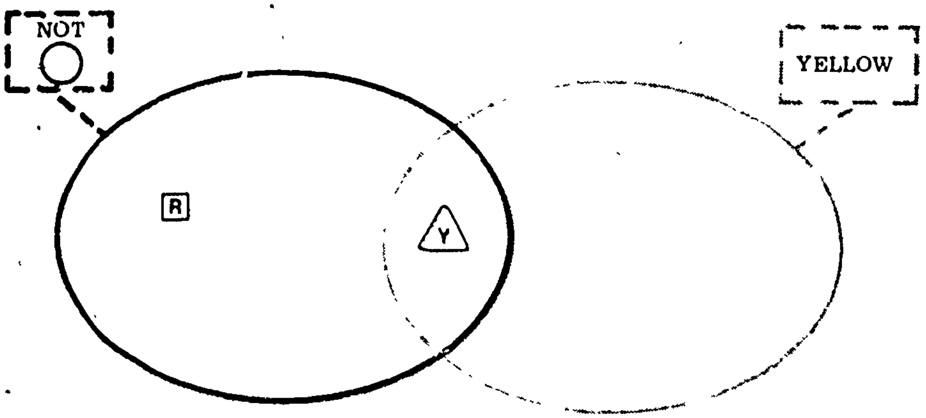


RED	YELLOW	GREEN	BLUE
NOT RED	NOT YELLOW	NOT GREEN	NOT BLUE
○	△	□	BIG
NOT ○	NOT △	NOT □	LITTLE

RED	YELLOW	GREEN	BLUE
NOT RED	NOT YELLOW	NOT GREEN	NOT BLUE
○	△	□	BIG
NOT ○	NOT △	NOT □	LITTLE

Proceed as in Activity S5. Discuss what information the big yellow triangle gives about the strings. Cross out attributes on the posters as students eliminate them from consideration. For example, NOT YELLOW and NOT △ may be eliminated from both charts since they would require the yellow triangle to be outside the string. Similarly, RED, GREEN, BLUE, ○ , □ , and LITTLE may be eliminated. Do not insist that students find all the attributes that can be crossed out if no more suggestions are forthcoming.

Give another clue.



After discussing each of the remaining possibilities for the labels, the charts will look like the following, if ultimate use is made of this clue.

Red String

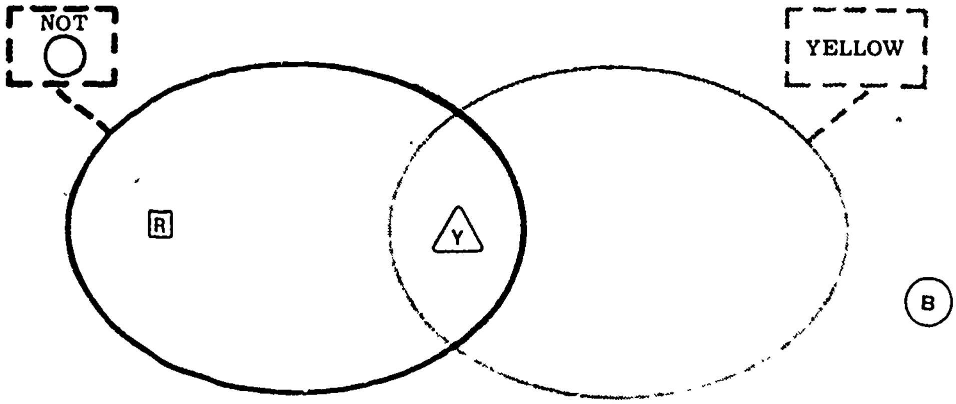
~~RED~~	~~YELLOW~~	~~GREEN~~	~~BLUE~~
~~NOT RED~~	~~NOT YELLOW~~	NOT GREEN	NOT BLUE
~~○~~	~~△~~	~~□~~	~~BIG~~
NOT ○	~~NOT △~~	~~NOT □~~	~~LITTLE~~

Blue String

~~RED~~	YELLOW	~~GREEN~~	~~BLUE~~
NOT RED	~~NOT YELLOW~~	~~NOT GREEN~~	~~NOT BLUE~~
~~○~~	△	~~□~~	BIG
~~NOT ○~~	~~NOT △~~	NOT □	~~LITTLE~~

ERIC
Full Text Provided by ERIC

Third Clue:



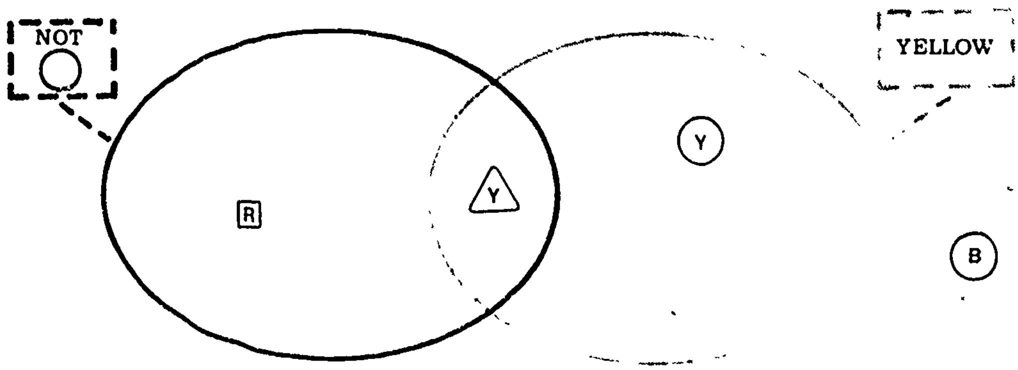
Again, use the clue to eliminate possibilities for the string labels.

~~RED~~	~~YELLOW~~	~~GREEN~~	~~BLUE~~
~~NOT RED~~	~~NOT YELLOW~~	~~NOT GREEN~~	NOT BLUE
~~○~~	~~△~~	~~□~~	~~BIG~~
NOT ○	~~NOT △~~	~~NOT □~~	~~LITTLE~~

~~RED~~	YELLOW	~~GREEN~~	~~BLUE~~
~~NOT RED~~	~~NOT YELLOW~~	~~NOT GREEN~~	~~NOT BLUE~~
~~○~~	△	~~□~~	~~BIG~~
~~NOT ○~~	~~NOT △~~	~~NOT □~~	~~LITTLE~~

After this clue, two possibilities remain for the red string--NOT BLUE and NOT ○ , and two possibilities are left for the blue string--YELLOW and △

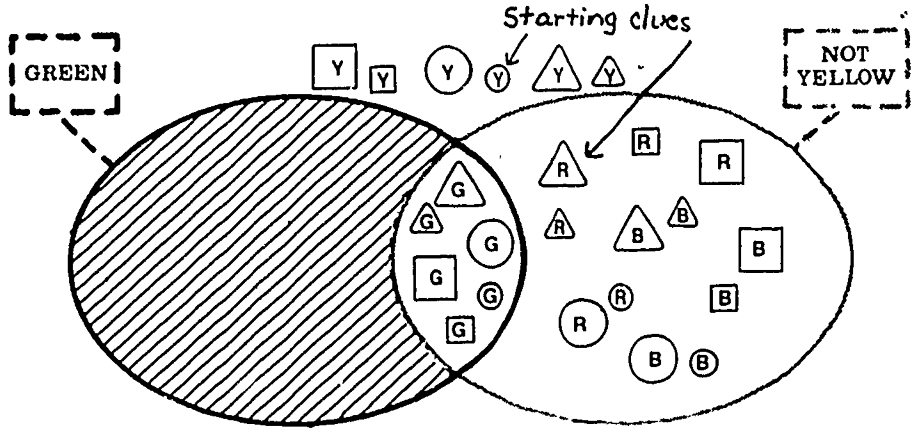
Fourth Clue:

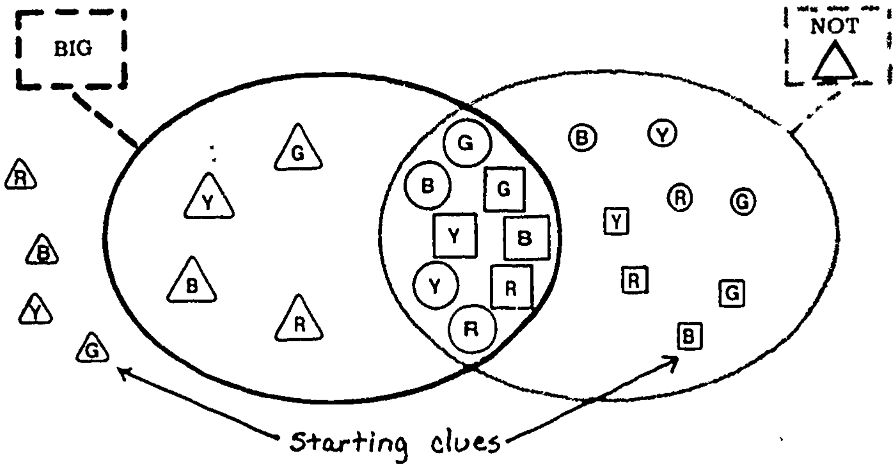


From this clue, your class can conclude:

- that the red string is labeled NOT ○ because NOT BLUE on the red string would require the yellow circle to be inside the red string; and
- that the blue string is labeled YELLOW because △ on the blue string would place the yellow circle outside the blue string.

Distribute analysis sheets found on page I-40 to the students. Play the String Game using not-cards. Before beginning the game, collectively analyze the starting clue as in the first clue above, then continue the game in the usual way. When pieces are placed correctly, encourage the students to cross out on their analysis sheets labels that the strings cannot have. Two of the many possible games are illustrated below.





ACTIVITY S8: THE STRING GAME WITH NUMBERS #1

PREREQUISITE: Activity S2

OBJECTIVE: Students will play the String Game with Numbers, using attributes of multiples and order.

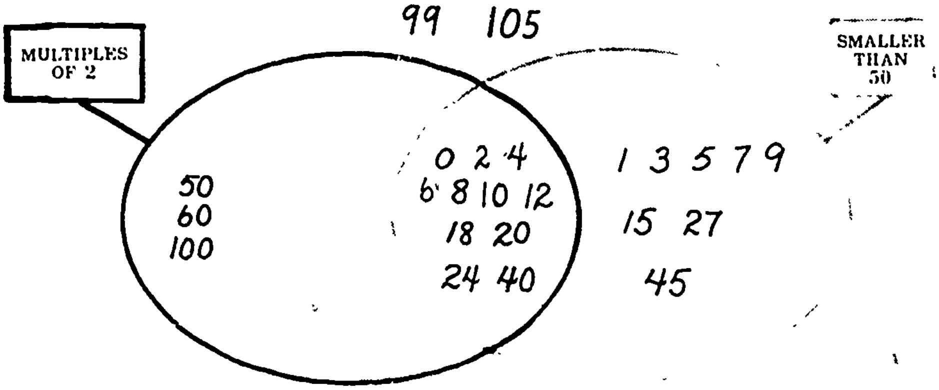
The String Game with Numbers is similar to the String Game with A-Blocks. The game is played with integers and attributes of integers. The game pieces and attributes for this version of the String Game are shown below.

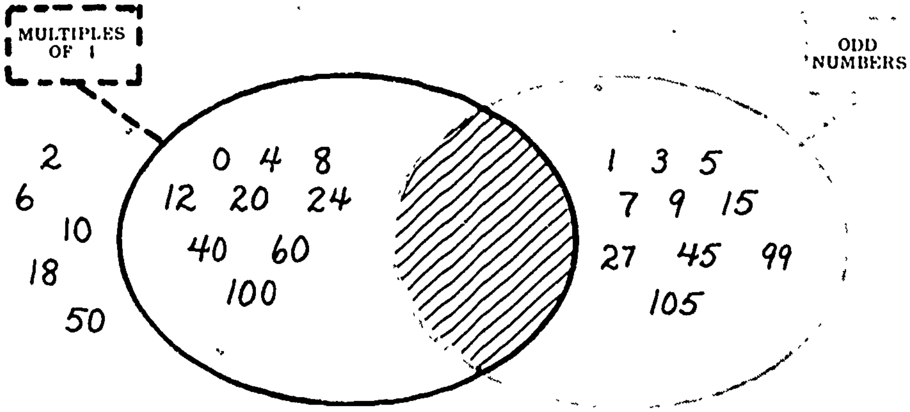
0	1	2	3	4	5
6	7	8	9	10	12
15	18	20	24	27	40
45	50	60	99	100	105

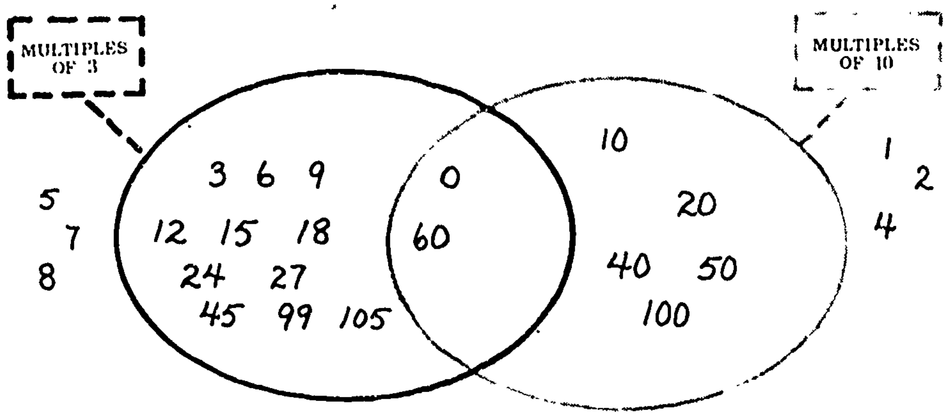
MULTIPLES OF 2	MULTIPLES OF 3	MULTIPLES OF 4	MULTIPLES OF 5
MULTIPLES OF 10	ODD NUMBERS	LARGER THAN 50	SMALLER THAN 50

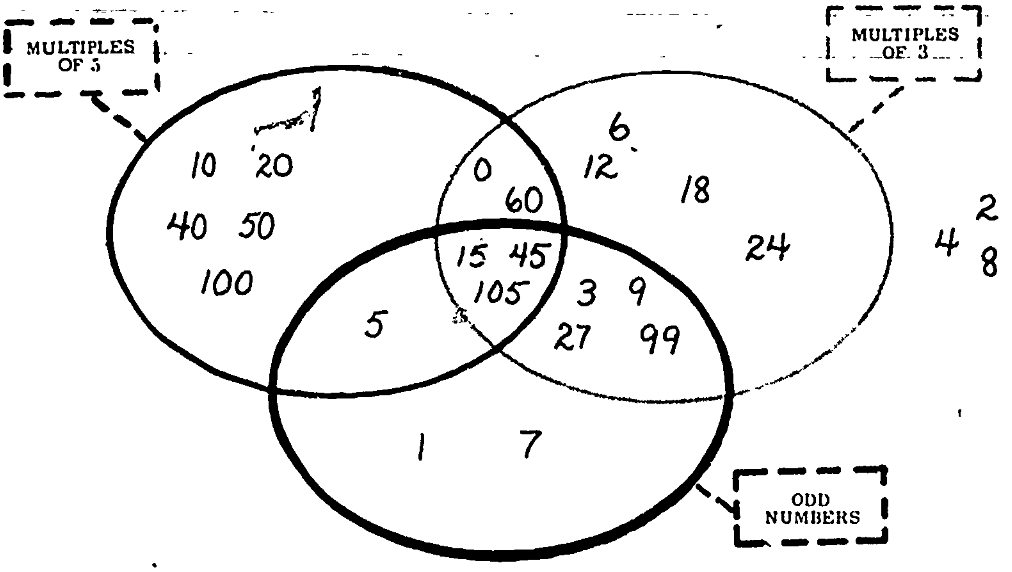
Note: Zero is a multiple of all numbers. If this conflicts with local curriculum, omit the "0" card from the game pieces, and play the game using only natural numbers. If you choose not to use "0", then remove "50" also to allow an even distribution of the pieces in a game.

The object and rules of the game are the same as those of the String Game with A-Blocks. To develop familiarity and understanding of the String Game with Numbers, play a few games with open labels as in Activities S1 and S6, before playing with the attribute cards face-down. Several examples of completed games are shown on the next two pages. Place two game pieces in the string picture before the game begins just as in playing the String Game with A-Blocks.

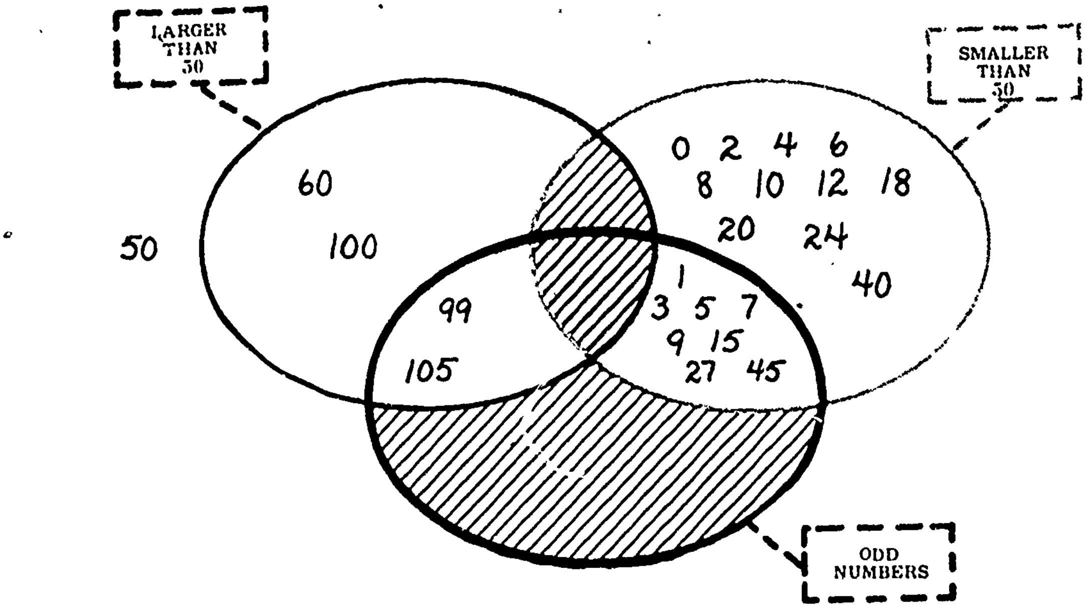

MULTIPLES OF 2
SMALLER THAN 50
99 105
50 60 100
0 2 4 6 8 10 12 18 20 24 40
1 3 5 7 9 15 27 45


MULTIPLES OF 4
ODD NUMBERS
2 6 10 18 50
0 4 8 12 20 24 40 60 100
1 3 5 7 9 15 27 45 99 105

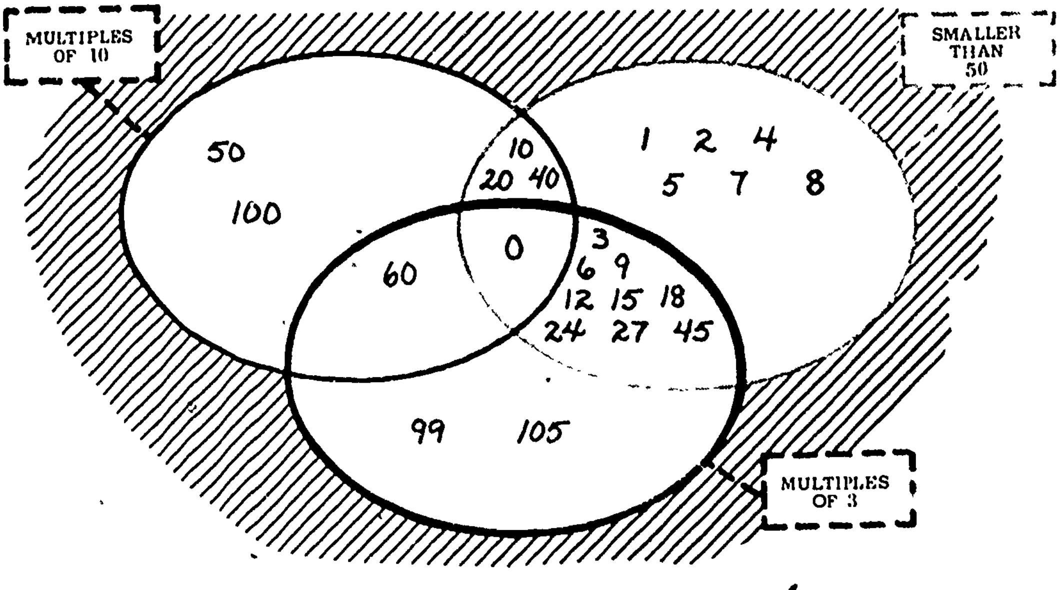

MULTIPLES OF 3
MULTIPLES OF 10
5 7 8
3 6 9 12 15 18 24 27 45 99 105
0 60
10 20 40 50 100
1 2 4



MULTIPLES OF 5
MULTIPLES OF 3
ODD NUMBERS
10 20 40 50 100
0 60
15 45 105
5
6 12 18 24
3 9 27 99
1 7
2 4 8



LARGER THAN 50
SMALLER THAN 50
ODD NUMBERS
50
60 100
0 2 4 6 8 10 12 18 20 24 40
99 105
1 3 5 7 9 15 27 45



MULTIPLES OF 10
SMALLER THAN 50
MULTIPLES OF 3
50 100
10 20 40
1 2 4 5 7 8
0
60
3 6 9 12 15 18 24 27 45
99 105

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ACTIVITY S9: THE STRING GAME WITH NUMBERS #2

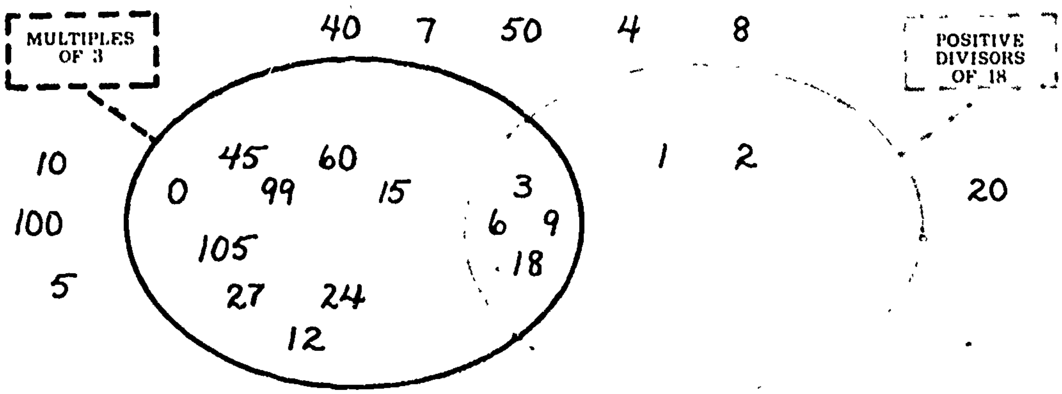
PREREQUISITE: Activity S8

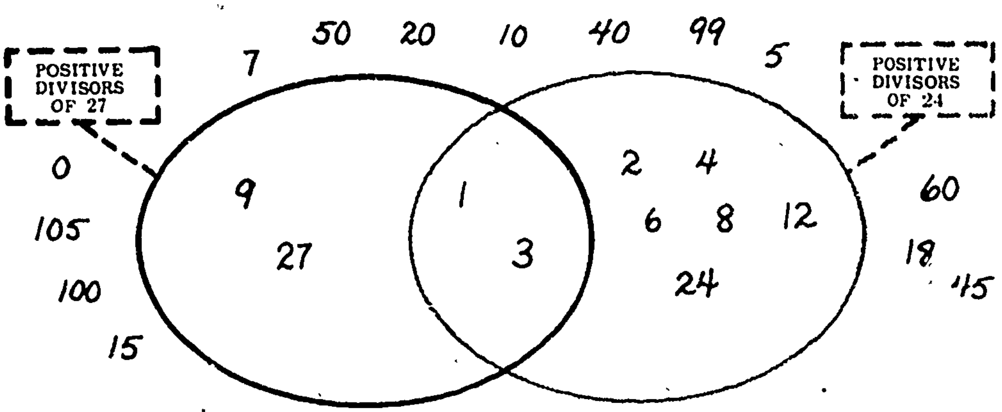
OBJECTIVE: Students will play the String Game with Numbers, using attributes of multiples, divisors, and order.

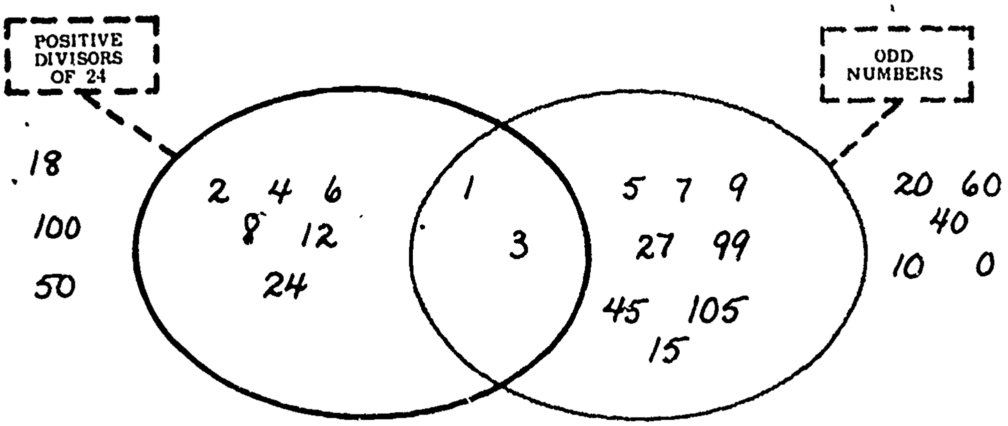
The attributes for this version of the game are shown in the following chart. Use the same game pieces as in S8.

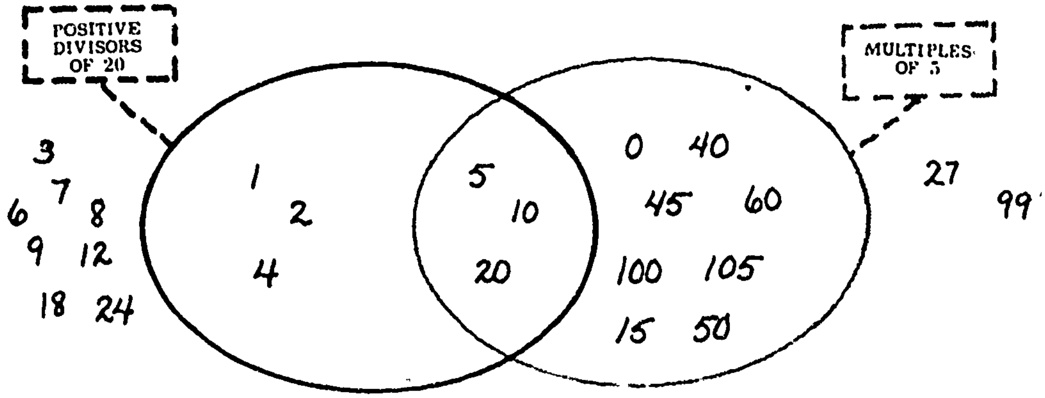
MULTIPLES OF 2	MULTIPLES OF 3	MULTIPLES OF 4
MULTIPLES OF 5	MULTIPLES OF 10	POSITIVE DIVISORS OF 12
POSITIVE DIVISORS OF 18	POSITIVE DIVISORS OF 20	POSITIVE DIVISORS OF 24
POSITIVE DIVISORS OF 27	LARGER THAN 50	SMALLER THAN 50
	ODD NUMBERS	

Several of the many possible games are illustrated below.




POSITIVE DIVISORS OF 27
POSITIVE DIVISORS OF 24
50 20 10 40 99 5 7
9 27
1 3
2 4 6 8 12 24
0 105 100 15
60 18 45


POSITIVE DIVISORS OF 24
ODD NUMBERS
2 4 6 8 12 24
1 3
5 7 9 27 99 45 105 15
18 100 50
20 60 40 10 0


POSITIVE DIVISORS OF 20
MULTIPLES OF 5
1 2 4
5 10 20
0 40 45 60 100 105 15 50
3 7 6 8 9 12 18 24
27 99

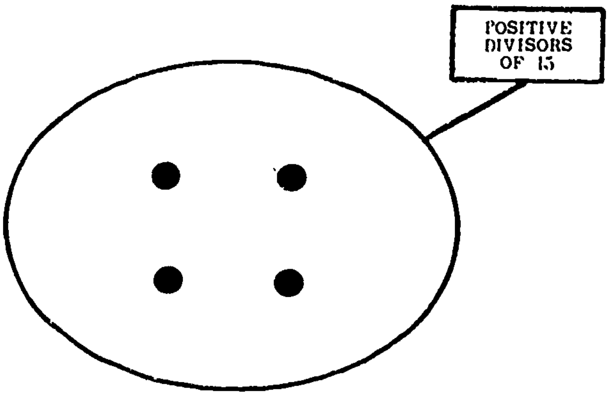
50

ACTIVITY S10: INTRODUCTION TO PRIME NUMBERS

PREREQUISITE: Activity S9

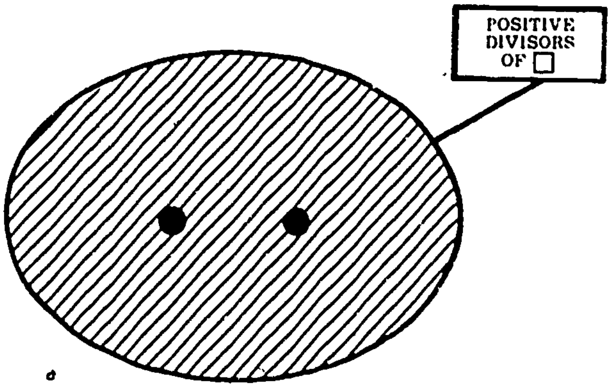
OBJECTIVE: Students will learn that prime numbers are integers that have exactly two positive divisors.

Draw the following string picture. Ask students to find the numbers that belong inside the string.

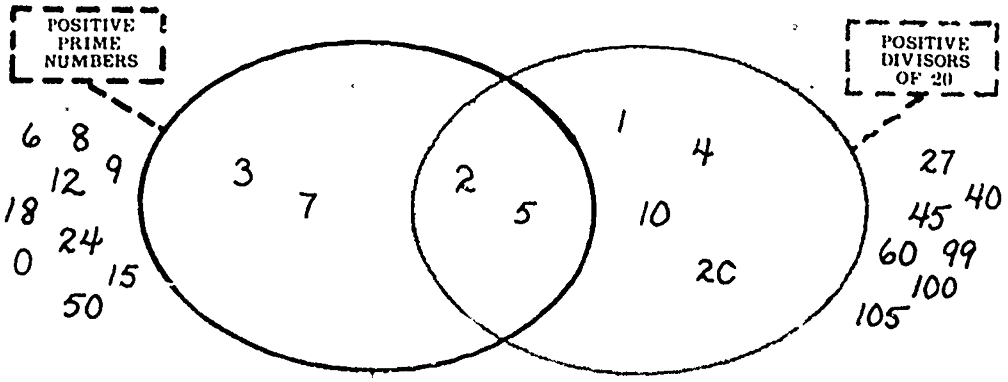


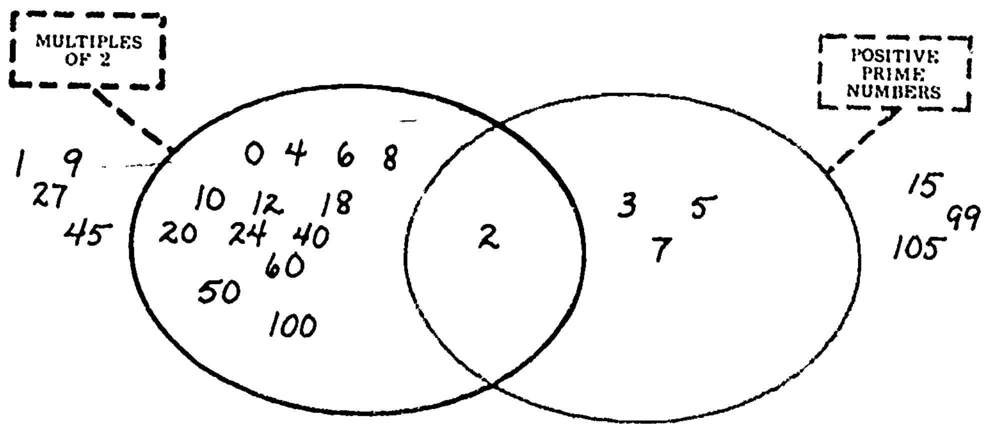
Erase the dots and labels and repeat this exercise, replacing "15" by "9" (three dots), "10" (four dots), "7" (two dots), "6", "4", "14", and "12."

Draw this string picture on the board. Ask students to find correct labels for the string and the dots. Mention that the hatching indicates there are exactly two numbers inside the string.



Integers that have exactly two positive divisors are called prime numbers. Ask the class to name the prime numbers already discovered and to find some more. The prime numbers less than 30 are 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29. This is a good attribute to use when playing the String Game. Two possible games are shown below.





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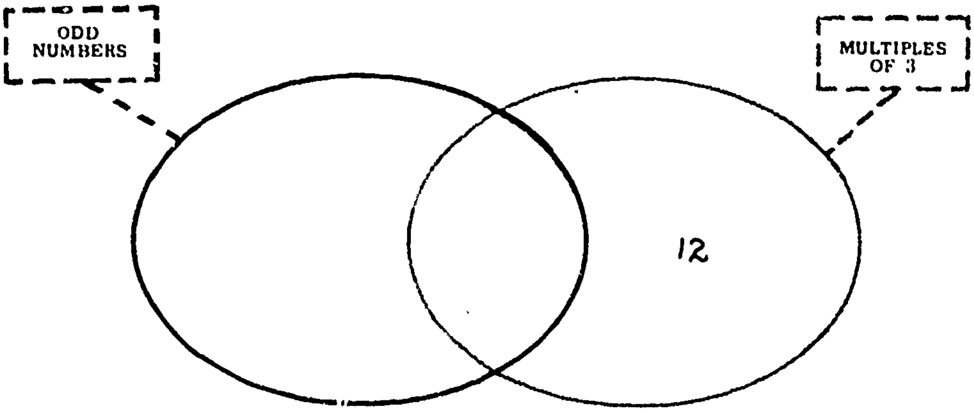
ACTIVITY S11: ANALYSIS OF THE STRING GAME WITH NUMBERS

PREREQUISITE: Activities S5 and S9

OBJECTIVE: Students will analyze the placement of numbers in string pictures to determine hidden attribute cards

Before the lesson begins, set up a String Game as illustrated below. Tape a demonstration analysis sheet on the board, one to the left of the red string and one to the right of the blue string.



~~MULTIPLES OF 2~~	~~MULTIPLES OF 3~~	~~MULTIPLES OF 4~~	MULTIPLES OF 5
MULTIPLES OF 10	~~POSITIVE DIVISORS OF 12~~	POSITIVE DIVISORS OF 18	POSITIVE DIVISORS OF 20
~~POSITIVE DIVISORS OF 24~~	POSITIVE DIVISORS OF 27	LARGER THAN 50	~~SMALLER THAN 50~~
	POSITIVE PRIME NUMBERS	ODD NUMBERS	

MULTIPLES OF 2	MULTIPLES OF 3	MULTIPLES OF 4	~~MULTIPLES OF 5~~
~~MULTIPLES OF 10~~	POSITIVE DIVISORS OF 12	~~POSITIVE DIVISORS OF 18~~	~~POSITIVE DIVISORS OF 20~~
POSITIVE DIVISORS OF 24	~~POSITIVE DIVISORS OF 27~~	~~LARGER THAN 50~~	SMALLER THAN 50
	~~POSITIVE PRIME NUMBERS~~	~~ODD NUMBERS~~	

Lead a discussion of the first clue. Since 12 is a multiple of 2, 3, and 4, a divisor of 24, and smaller than 50, all of these labels are eliminated as possibilities for the red string's attribute card. Since 12 is not a multiple of 5 or 10 and not a divisor of 18, 20, or 27, none of these can be the blue string's attribute card. Similarly LARGER THAN 50, ODD NUMBERS, and POSITIVE PRIME NUMBERS, are all eliminated for the blue string.

Second clue: Place "20" outside both strings. This clue eliminates MULTIPLES OF 5, MULTIPLES OF 10, and POSITIVE DIVISORS of 20 for the red string's attribute card, and MULTIPLES OF 2, MULTIPLES OF 4, and SMALLER THAN 50 for the blue string's attribute card.

Third clue: Place "45" inside both strings. Encourage a full discussion of this clue until students determine that the red strings attribute card is ODD NUMBERS and the blue strings attribute card is MULTIPLES OF 3.

Reproduce and distribute String Game analysis sheets found on page I-41.

T: Now we will play a String Game. Each of you has a chart of the possible labels for the red string and the blue string. Use this chart during the game to help you determine the hidden labels. Cross out attributes that are eliminated by clues.

Begin with two clues. Since students are new to analysis, lead a collective discussion of the first clue; in other activities, let them work independently. Afterward let players take turns placing pieces as they did in the string game. Give the students time to analyze plays by pausing briefly between turns when a player has placed a piece correctly. Such pauses in the game encourage students to use the clues in crossing out labels on their charts.

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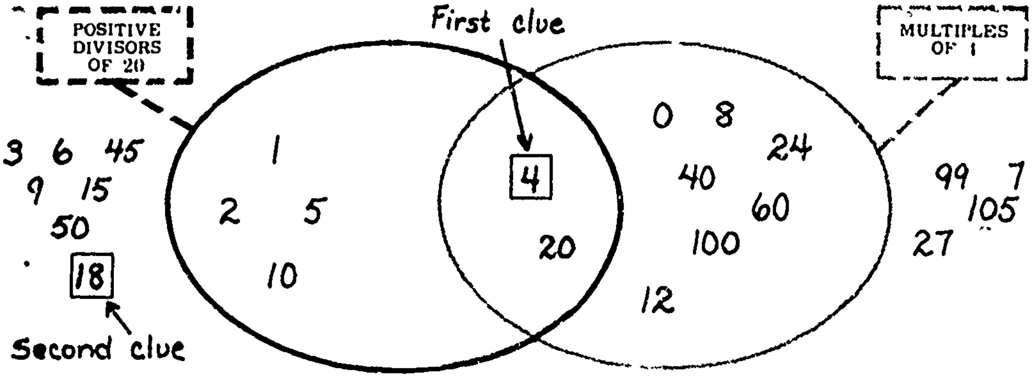
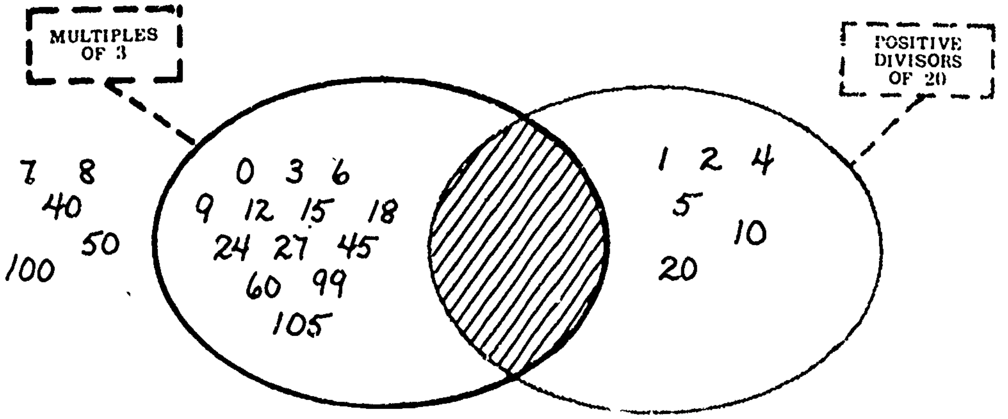
When a piece is played incorrectly, record that information in the string picture. For example, if "27" is played incorrectly in the center region, replace the piece on the teamboard and write "~~27~~" in the center region.

Several of the many possible games are shown below.





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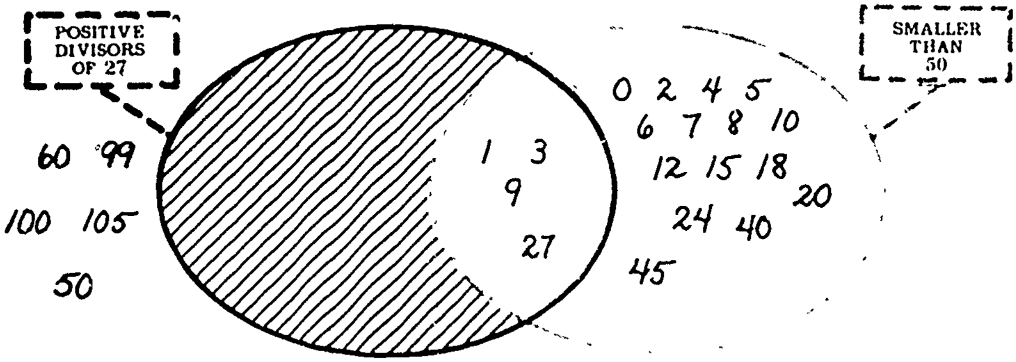
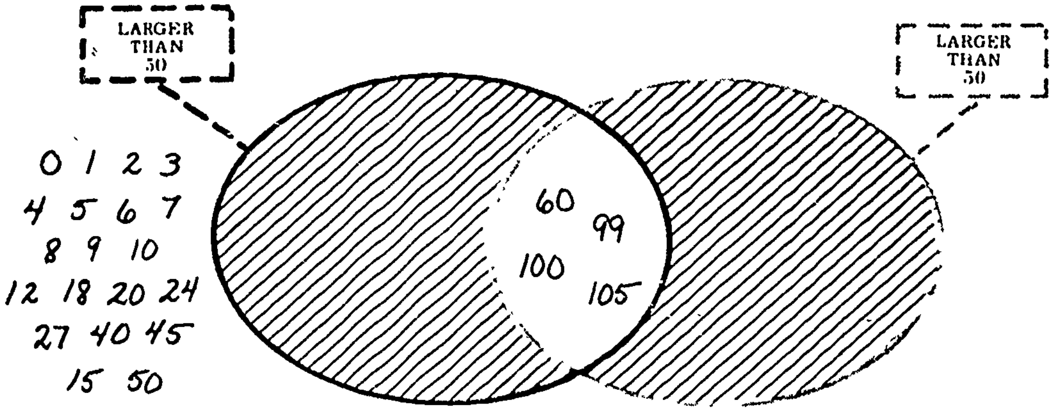
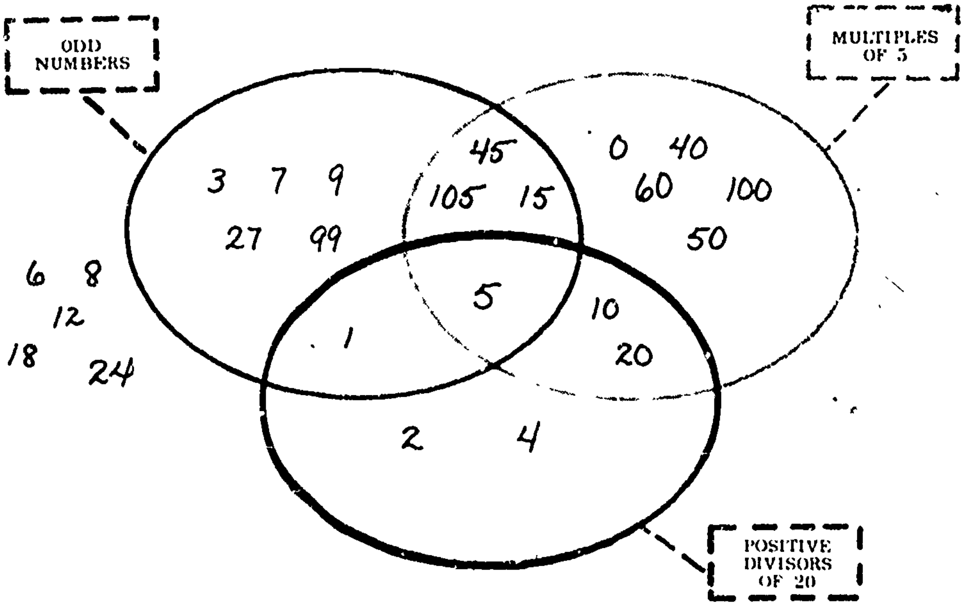
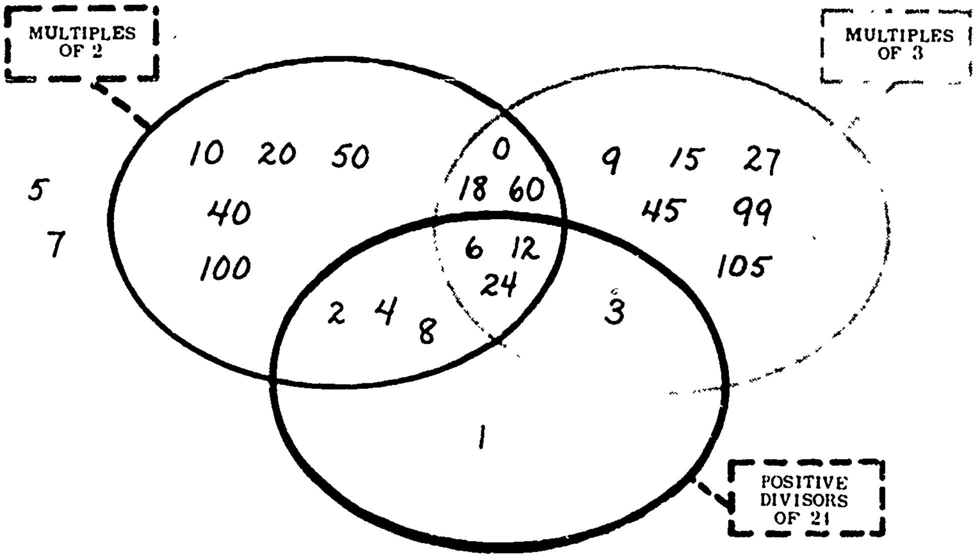
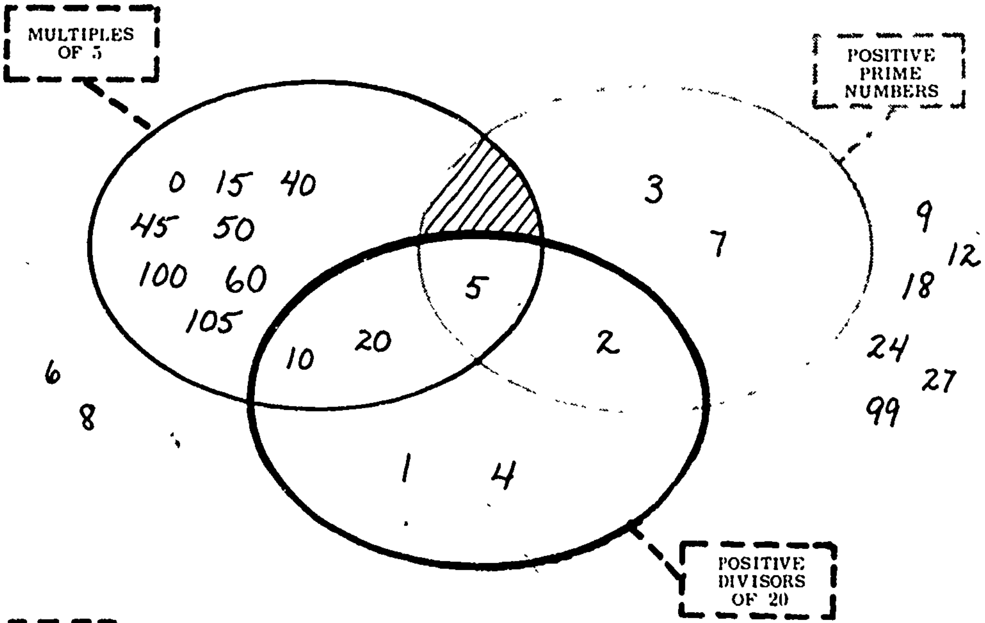

POSITIVE DIVISORS OF 27
SMALLER THAN 50
1 3 9 27
0 2 4 5 6 7 8 10 12 15 18 20 24 40 45
60 99 100 105 50

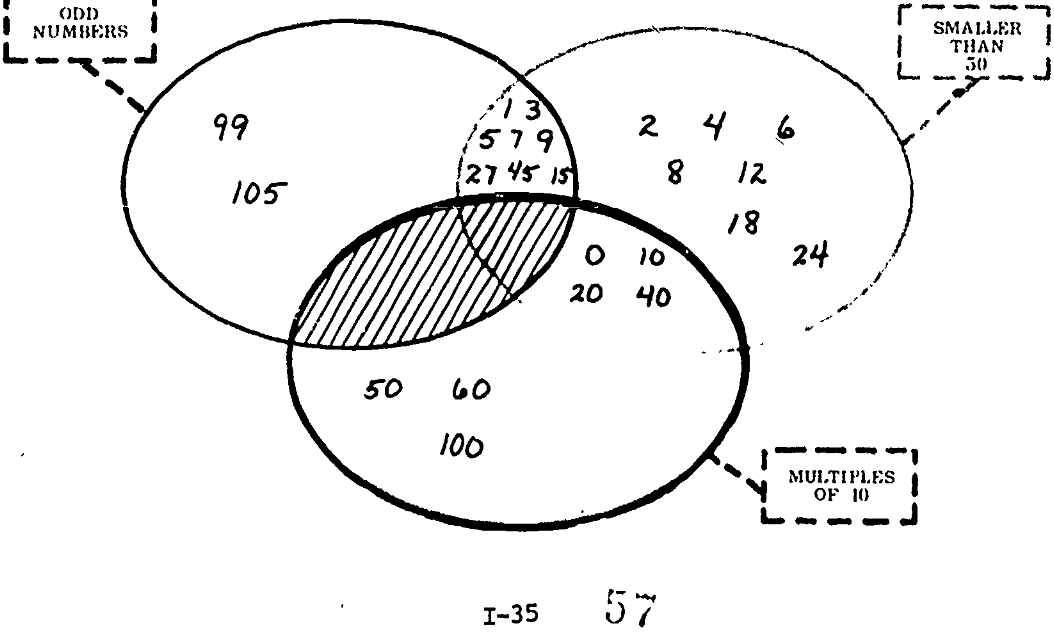

LARGER THAN 50
LARGER THAN 50
60 99 100 105
0 1 2 3 4 5 6 7 8 9 10 12 18 20 24 27 40 45 15 50


ODD NUMBERS
MULTIPLES OF 5
POSITIVE DIVISORS OF 20
3 7 9 27 99
45 105 15
0 40 60 100 50
5
1
10 20
2 4
6 8 12 18 24

56


MULTIPLES OF 2
MULTIPLES OF 3
POSITIVE DIVISORS OF 24
10 20 50
40
100
0
18 60
9 15 27
45 99
105
6 12
24
2 4 8
3
1
5
7


MULTIPLES OF 5
POSITIVE PRIME NUMBERS
POSITIVE DIVISORS OF 20
0 15 40
45 50
100 60
105
3
7
5
10 20
2
1 4
6
8
9
12
18
24
27
99


ODD NUMBERS
SMALLER THAN 30
MULTIPLES OF 10
99
105
1 3
5 7 9
27 45 15
2 4 6
8 12
18
24
0 10
20 40
50 60
100

ACTIVITY S12: THE STRING GAME WITH NUMBERS #3

PREREQUISITE: Activity S9

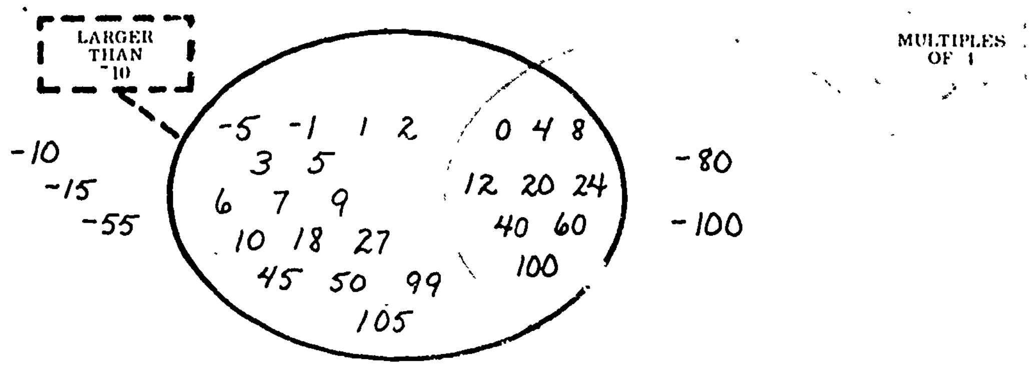
OBJECTIVE: Students will play a version of the String Game that includes negative numbers.

The following are the game pieces and attributes for this version of the String Game.

-100	-80	-55	-15	-10	-5
-1	0	1	2	3	4
5	6	7	8	9	10
12	18	20	24	27	40
45	50	60	99	100	105

MULTIPLES OF 2	MULTIPLES OF 3	MULTIPLES OF 4	MULTIPLES OF 5
MULTIPLES OF 10	POSITIVE DIVISORS OF 12	POSITIVE DIVISORS OF 18	POSITIVE DIVISORS OF 20
POSITIVE DIVISORS OF 24	POSITIVE DIVISORS OF 27	LARGER THAN 50	LARGER THAN -10
SMALLER THAN 50	SMALLER THAN -10	ODD NUMBERS	POSITIVE PRIME NUMBERS

Some of the many possible games are illustrated here.



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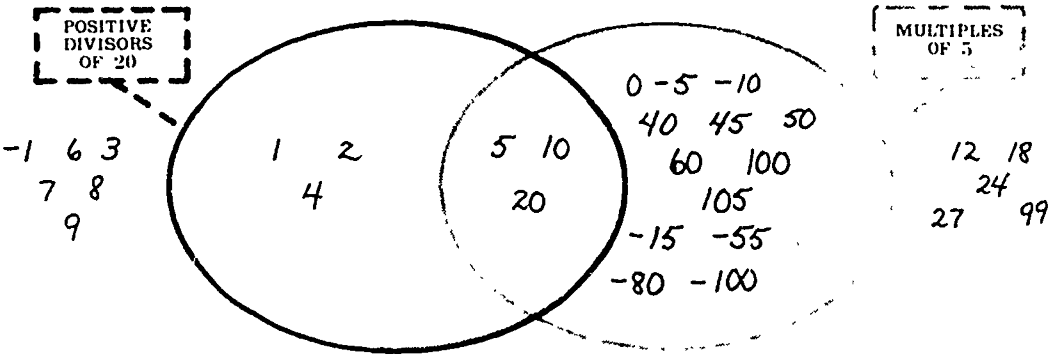
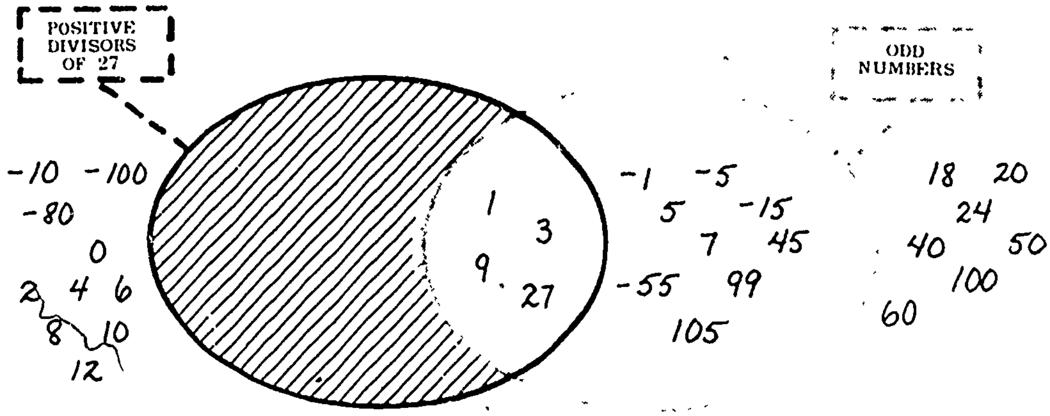
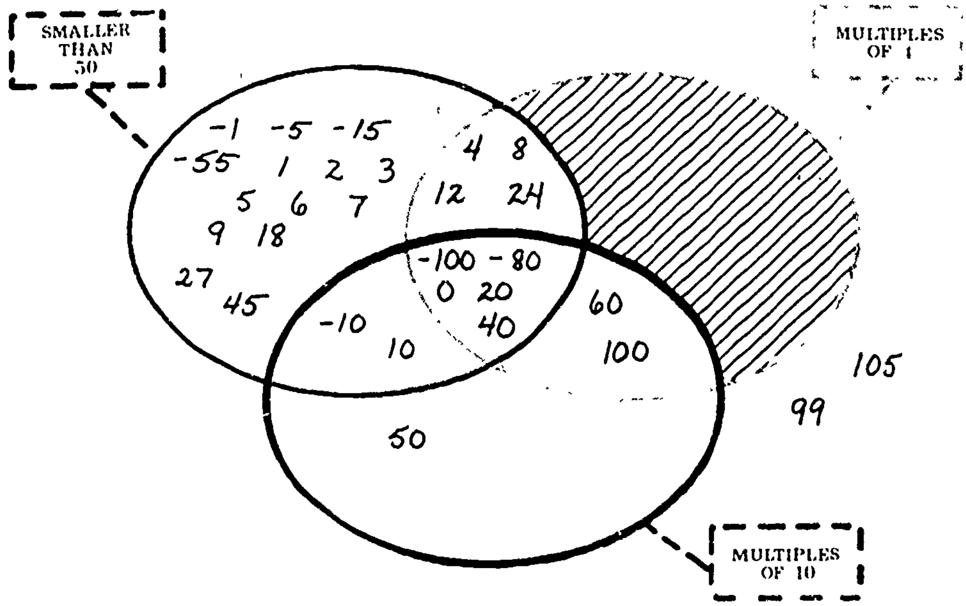

POSITIVE DIVISORS OF 20
MULTIPLES OF 5
-1 6 3
7 8
9
1 2
4
5 10
20
0 -5 -10
40 45 50
60 100
105
-15 -55
-80 -100
12 18
24
27 99


POSITIVE DIVISORS OF 27
ODD NUMBERS
-10 -100
-80
0
2 4 6
8 10
12
1
3
9
27
-1 -5
5 -15
7 45
-55 99
105
18 20
24
40 50
100
60


SMALLER THAN 50
MULTIPLES OF 4
-1 -5 -15
-55 1 2 3
5 6 7
9 18
27
45
4 8
12 24
-100 -80
0 20
40
-10
10
60
100
50
105
99
MULTIPLES OF 10

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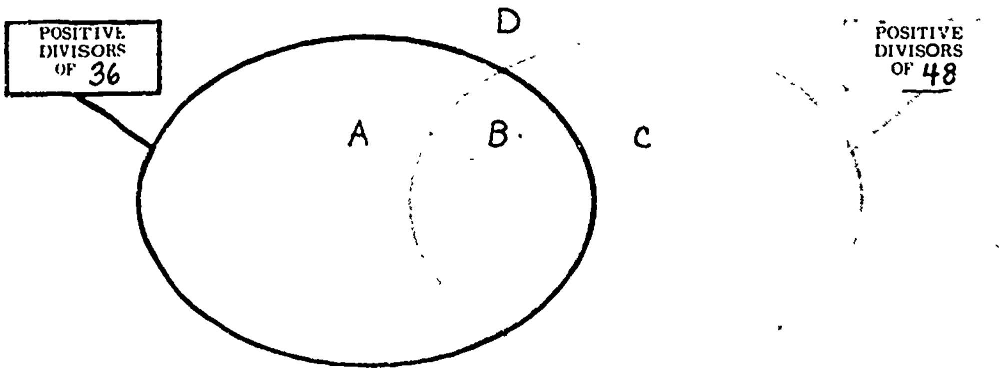
ACTIVITY S13: THE DIVISOR GAME

PREREQUISITE: Activity S9

OBJECTIVE: Students will develop their knowledge of common divisors.

Draw this string picture on the board.



Let students suggest and place numbers in the string picture. Discuss each suggestion and allow correct suggestions to remain in the picture. For example, suppose "9" is placed in region A.

T: Let's see if "9" is correctly placed. Is 9 a positive divisor of 36? (Yes) Why? (9 x 4 = 36) Is 9 a positive divisor of 48? (No) Why? (48 is not a multiple of 9.) So "9" belongs in region A.

Occasionally point to a region and ask for a number that could be placed in that region. Occasionally select a number (e.g, 8) and ask a student to place it in the correct region and justify the placement. Select numbers and students so that all students are encouraged to participate. There are numbers and questions appropriate for weak as well as for strong students.

Repeat the exercise with the positive divisors of 18 and 24, 12 and 24, 6 and 8, and other appropriate number pairs.

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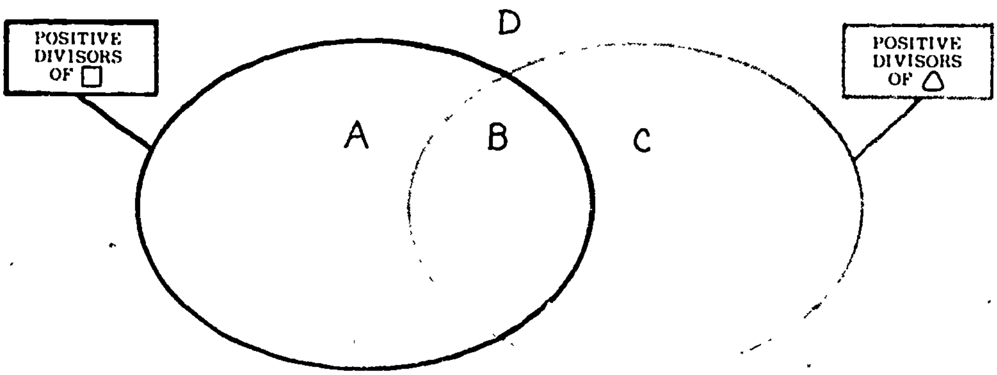
Draw the following string picture on the board.



T: The red string contains all positive divisors of a secret number we will call "box". The blue string contains all positive divisors of a number we will call "wedge". Your task is to determine box and wedge. Each of them is a whole number less than or equal to 50. Where does "1" belong in this picture? (In region B, since 1 is a divisor of all whole numbers)

Let the class play freely, with no expectation of analysis. On a turn, a player may either select and attempt to correctly place any whole number from 2 to 50 in the string picture or guess either box or wedge. The game ends when both attribute cards have been identified correctly. Invite students to confirm whether suggestions for box and wedge are correct. When both attribute cards are identified, ask students to place more numbers in their correct regions.

You can play this game with any pair of numbers. If appropriate to your curriculum, this game can easily be related to the study of common divisors and greatest common divisors.

When the students are familiar with the Divisors Game, divide the class into two teams and play competitively. Devise a scoring system that will determine which team wins the game.

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ANALYSIS SHEETS

MULTIPLES OF 2	MULTIPLES OF 3	MULTIPLES OF 4	MULTIPLES OF 5
MULTIPLES OF 10	ODD NUMBERS	LARGER THAN 50	SMALLER THAN 50

MULTIPLES OF 2	MULTIPLES OF 3	MULTIPLES OF 4	MULTIPLES OF 5
MULTIPLES OF 10	ODD NUMBERS	LARGER THAN 50	SMALLER THAN 50

MULTIPLES OF 2	MULTIPLES OF 3	MULTIPLES OF 4	MULTIPLES OF 5
MULTIPLES OF 10	ODD NUMBERS	LARGER THAN 50	SMALLER THAN 50

MULTIPLES OF 2	MULTIPLES OF 3	MULTIPLES OF 4	MULTIPLES OF 5
MULTIPLES OF 10	ODD NUMBERS	LARGER THAN 50	SMALLER THAN 50

RED	YELLOW	GREEN	BLUE
NOT RED	NOT YELLOW	NOT GREEN	NOT BLUE
○	△	□	BIG
NOT ○	NOT △	NOT □	LITTLE

RED	YELLOW	GREEN	BLUE
NOT RED	NOT YELLOW	NOT GREEN	NOT BLUE
○	△	□	BIG
NOT ○	NOT △	NOT □	LITTLE

RED	YELLOW	GREEN	BLUE
NOT RED	NOT YELLOW	NOT GREEN	NOT BLUE
○	△	□	BIG
NOT ○	NOT △	NOT □	LITTLE

	BIG	LITTLE	
○	△	□	
RED	YELLOW	GREEN	BLUE

	BIG	LITTLE	
○	△	□	
RED	YELLOW	GREEN	BLUE

	BIG	LITTLE	
○	△	□	
RED	YELLOW	GREEN	BLUE

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MULTIPLES OF 2	MULTIPLES OF 3	MULTIPLES OF 4	MULTIPLES OF 5
MULTIPLES OF 10	POSITIVE DIVISORS OF 12	POSITIVE DIVISORS OF 18	POSITIVE DIVISORS OF 20
POSITIVE DIVISORS OF 24	POSITIVE DIVISORS OF 27	LARGER THAN 50	LARGER THAN $^{-}10$
SMALLER THAN 50	SMALLER THAN $^{-}10$	ODD NUMBERS	POSITIVE PRIME NUMBERS

MULTIPLES OF 2	MULTIPLES OF 3	MULTIPLES OF 4	MULTIPLES OF 5
MULTIPLES OF 10	POSITIVE DIVISORS OF 12	POSITIVE DIVISORS OF 18	POSITIVE DIVISORS OF 20
POSITIVE DIVISORS OF 24	POSITIVE DIVISORS OF 27	LARGER THAN 50	SMALLER THAN 50
	POSITIVE PRIME NUMBERS	ODD NUMBERS	

MULTIPLES OF 2	MULTIPLES OF 3	MULTIPLES OF 4	MULTIPLES OF 5
MULTIPLES OF 10	POSITIVE DIVISORS OF 12	POSITIVE DIVISORS OF 18	POSITIVE DIVISORS OF 20
POSITIVE DIVISORS OF 24	POSITIVE DIVISORS OF 27	LARGER THAN 50	LARGER THAN $^{-}10$
SMALLER THAN 50	SMALLER THAN $^{-}10$	ODD NUMBERS	POSITIVE PRIME NUMBERS

MULTIPLES OF 2	MULTIPLES OF 3	MULTIPLES OF 4	MULTIPLES OF 5
MULTIPLES OF 10	POSITIVE DIVISORS OF 12	POSITIVE DIVISORS OF 18	POSITIVE DIVISORS OF 20
POSITIVE DIVISORS OF 24	POSITIVE DIVISORS OF 27	LARGER THAN 50	SMALLER THAN 50
	POSITIVE PRIME NUMBERS	ODD NUMBERS	

MULTIPLES OF 2	MULTIPLES OF 3	MULTIPLES OF 4	MULTIPLES OF 5
MULTIPLES OF 10	POSITIVE DIVISORS OF 12	POSITIVE DIVISORS OF 18	POSITIVE DIVISORS OF 20
POSITIVE DIVISORS OF 24	POSITIVE DIVISORS OF 27	LARGER THAN 50	LARGER THAN $^{-}10$
SMALLER THAN 50	SMALLER THAN $^{-}10$	ODD NUMBERS	POSITIVE PRIME NUMBERS

MULTIPLES OF 2	MULTIPLES OF 3	MULTIPLES OF 4	MULTIPLES OF 5
MULTIPLES OF 10	POSITIVE DIVISORS OF 12	POSITIVE DIVISORS OF 18	POSITIVE DIVISORS OF 20
POSITIVE DIVISORS OF 24	POSITIVE DIVISORS OF 27	LARGER THAN 50	SMALLER THAN 50
	POSITIVE PRIME NUMBERS	ODD NUMBERS	

ANALYSIS SHEETS

MULTIPLES OF 2	MULTIPLES OF 3	MULTIPLES OF 4
MULTIPLES OF 5	MULTIPLES OF 10	POSITIVE DIVISORS OF 12
POSITIVE DIVISORS OF 18	POSITIVE DIVISORS OF 20	POSITIVE DIVISORS OF 24
POSITIVE DIVISORS OF 27	LARGER THAN 50	SMALLER THAN 50
	ODD NUMBERS	

MULTIPLES OF 2	MULTIPLES OF 3	MULTIPLES OF 4
MULTIPLES OF 5	MULTIPLES OF 10	POSITIVE DIVISORS OF 12
POSITIVE DIVISORS OF 18	POSITIVE DIVISORS OF 20	POSITIVE DIVISORS OF 24
POSITIVE DIVISORS OF 27	LARGER THAN 50	SMALLER THAN 50
	ODD NUMBERS	

MULTIPLES OF 2	MULTIPLES OF 3	MULTIPLES OF 4
MULTIPLES OF 5	MULTIPLES OF 10	POSITIVE DIVISORS OF 12
POSITIVE DIVISORS OF 18	POSITIVE DIVISORS OF 20	POSITIVE DIVISORS OF 24
POSITIVE DIVISORS OF 27	LARGER THAN 50	SMALLER THAN 50
	ODD NUMBERS	

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STRAND II: PROBLEM SOLVING WITH THE MINICOMPUTER (Continued)

Negative Numbers

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STRAND II: PROBLEM SOLVING WITH THE MINICOMPUTER (Continued)

STRAND II: PROBLEM SOLVING WITH THE MINICOMPUTER (Continued)

Decimal Numbers

ERIC

INTRODUCTION

The Papy Minicomputer, invented by the Belgian mathematician Georges Papy, is essentially an abacus that models the positional structure of our customary system of numeration. As such, it provides a powerful tool for arithmetic calculation and, moreover, a rich vein of situations for numerical investigations. It is not a sophisticated electronic device, but rather it consists of one or more boards, each board subdivided into four squares, and a set of markers or checkers. When one or more Minicomputer boards are displayed side-by-side, the position that each board holds relative to the other boards corresponds to customary decimal place value (ones, tens, hundreds, and so on). Often colored for pedagogical convenience as indicated, the four squares of a board confer value on any resident checker according to the board's relative position as follows:

BROWN	PURPLE
RED	WHITE

- white - 1, 10, 100, ...
- red - 2, 20, 200, ...
- purple - 4, 40, 400, ...
- brown - 8, 80, 800, ...

The number represented by a configuration of checkers on the Minicomputer is the sum of the values of all of the checkers on the boards. Of course, a number may have many different Minicomputer configurations. The Minicomputer representation of a negative number employs checkers with "^" written on them according to the convention that such checkers receive the negative of the value ordinarily conferred by a square. Some examples of configurations are shown on the next page.

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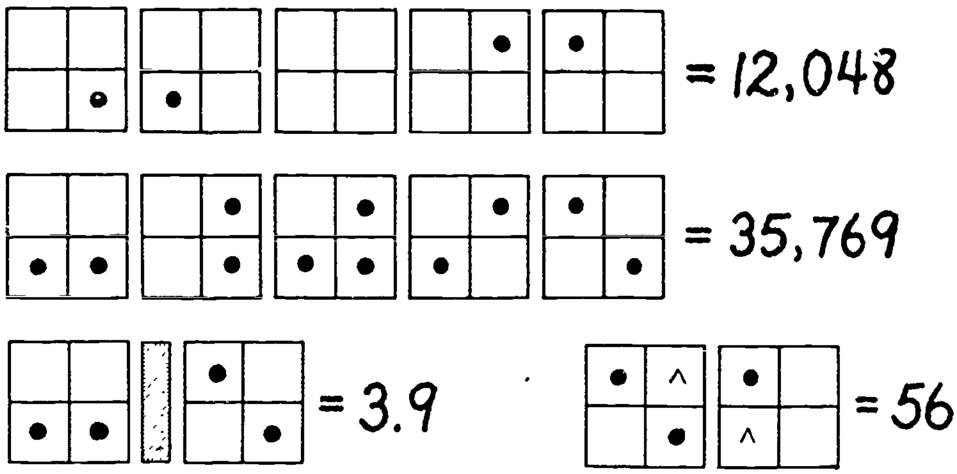


Subject to extensive study and experimentation since its invention thirty years ago, the Minicomputer is available for a wide range of purposes and abilities. For students at an early stage of numerical development, the Minicomputer provides a means for becoming familiar even with relatively large numbers and very small numbers through the convenient representation of place values. The power of the Minicomputer only becomes apparent in examining its varying role over the course of the arithmetic syllabus such as in its use in the CSMP curriculum. In the beginning, it serves primarily as a tool for exploring numbers, their interrelationships and their anatomy through the variety of configurations available. As students progress, the Minicomputer is important less as a tool for calculation and more as a device to stimulate mental arithmetic, to pose challenging problems about numbers, and to encourage creative thinking about the nature and properties of numbers.

The activities in this strand are divided into three sections: Whole Numbers (W), Decimal Numbers (D), and Negative Numbers (N). Introduce your students to the Minicomputer by presenting Activities W1 to W13. Continue with additional activities drawn from the rest of the strand, paying attention to the stated prerequisites. Arrange a schedule to fit your students' abilities and needs.

View the activities presented here not as complete lessons, but rather as sources of ideas and sample problems. To expand a lesson according to your students' abilities, select problems from one or more activities and create additional similar problems. The lessons can offer a mixture of whole group, small-group, and individual problem solving.

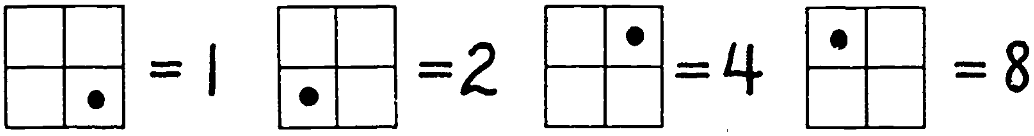
Everyone's facility with the Minicomputer grows with its use. The time initially spent in learning to use it will be repaid in pleasure and growth through later playing Minicomputer games and solving Minicomputer problems.

ACTIVITY W1: INTRODUCTION TO THE MINICOMPUTER #1

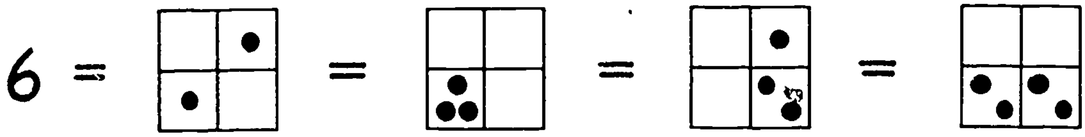
PREREQUISITE: None

OBJECTIVE: Students will identify and put numbers on the ones' board of the Minicomputer.

Display one Minicomputer board. Move one checker from square to square showing how to represent 1, 2, 4, and 8 on the Minicomputer.

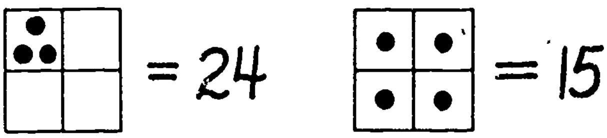


Challenge students to show 6 on the Minicomputer. Encourage several solutions. For example,



For each configuration, ask students, "Why is this 6?" Explanations may involve addition (4 + 2 = 6) or multiplication (3 x 2 = 6). Ask students to show other numbers--for example, 9, 13, 5, 20.

Put a number on the Minicomputer and ask a student to identify it. For example,



Select several more numbers for students to identify. If you wish, ask students to write each answer down so it can be quickly checked before a student answers aloud.

ACTIVITY W2: COMBINATORIAL PROBLEMS #1

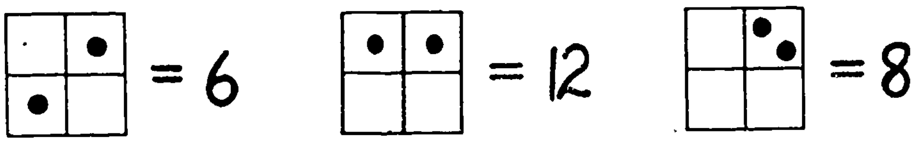
PREREQUISITE: Activity W1

OBJECTIVE: Students will list all the numbers that can be put on the ones' board of the Minicomputer with exactly two checkers.

Display one Minicomputer board.

T: What numbers can be put on this Minicomputer with exactly two checkers?

Let students put numbers on the Minicomputer. For example,



T: What is the largest number that can be put on the Minicomputer with exactly two checkers? (16)
What is the smallest number that can be put on the Minicomputer with exactly two checkers? (2)
What other numbers between 2 and 16 can be put on the Minicomputer with exactly two checkers? (3, 4, 5, 6, 8, 9, 10, 12) Let's list them.

As numbers are suggested, ask students to put them on the Minicomputer.

T: Some of the numbers between 2 and 16 are missing from our list. Why?

Let students discuss this. One explanation is that the two largest possible numbers are 8 + 8 = 16 and 8 + 4 = 12; so 13, 14, and 15 are all impossible to represent on one Minicomputer board with exactly two checkers.

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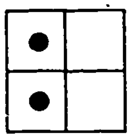
ACTIVITY W3: MISCELLANEOUS PROBLEMS #1

PREREQUISITE: Activity W2

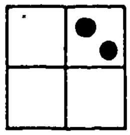
OBJECTIVE: Students working individually or in small groups will put a number on the Minicomputer using a specified number of checkers.

Display one Minicomputer board. Present the following problems or similar problems to the class.

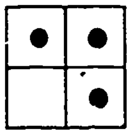
T: Put 10 on the Minicomputer with exactly two checkers.



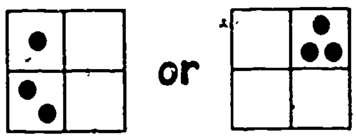
Put on 8 with exactly two checkers.



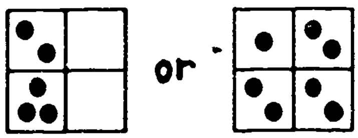
Put on 13 with exactly three checkers.



Put on 12 with exactly three checkers. Can you do it another way?



Put on 22 with exactly five checkers. (There are several solutions.)



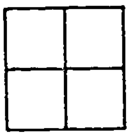
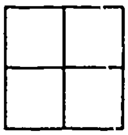
Divide the class into four groups. Give each group one demonstration Minicomputer board and five magnetic checkers. Write the following problems on the board. The members of each group should work together to solve the problems. Check each solution as a group solves a problem.

Two Checkers: 12, 16 Three Checkers: 11, 8

Four Checkers: 26, 10 Five Checkers: 30, 21

If you wish, put these problems on a worksheet, using the Minicomputers on page 93. Give one copy of the worksheet to each group. For example, the first two problems can be:

> Using exactly two checkers, put these numbers on the Minicomputer. Draw dots to show your answers.

 = 12  = 16

ACTIVITY W4: MINICOMPUTER NIM #1

PREREQUISITE: Activity W1

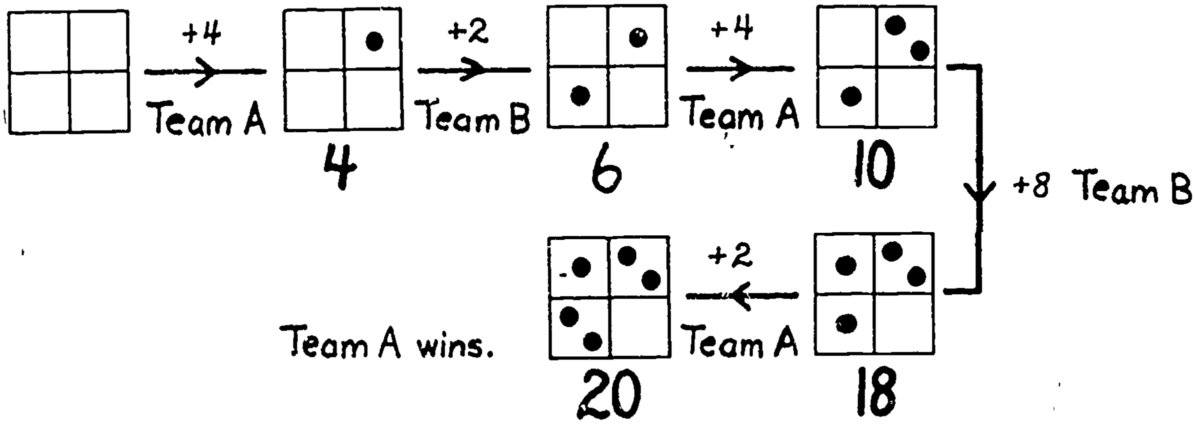
OBJECTIVE: Students will learn to play a competitive game whose goal is to put a specific number on the Minicomputer.

Display a Minicomputer board. Divide the class equally into two teams, Team A and Team B. Write "GOAL: 20" on the board.

T: What number is on the Minicomputer? (0) I will show you a game. In this game, we start at 0 and put one checker at a time on the Minicomputer until we reach another number. Today we'll try to put 20 on the Minicomputer. We play with two teams, Team A and Team B. The two teams take turns. On your turn you put one checker on the Minicomputer. You are not allowed to go over 20. The first team to reach 20 wins.

Invite a student from Team A to place the first checker.

An illustration of a short game follows.



Play the game several times. If students enjoy the game, play it frequently as a warm-up activity or when you have a few extra minutes. Vary the game by increasing the goal and, eventually, by playing with several Minicomputer boards.

ACTIVITY W5: INTRODUCTION TO THE MINICOMPUTER #2

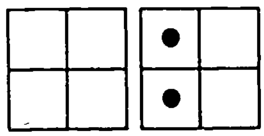
PREREQUISITE: Activity W1

OBJECTIVE: Students will be able to identify and put numbers on the ones' and tens' boards of the Minicomputer.

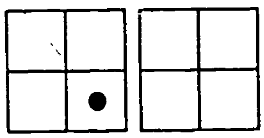
Display one Minicomputer board. One at a time, put the numbers 1, 2, 4, 8, 6, 9, 10, 7 on the Minicomputer and ask students to identify them. You may wish to ask students to record each number on a piece of paper.

Invite students to put the numbers 5, 3, 16, 13 on the Minicomputer.

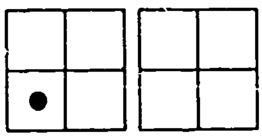
Display two Minicomputer boards, side by side, and place two checkers as follows.



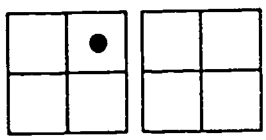
T: What number is this? (10) I can put 10 on the Minicomputer with just one checker. This is 10.



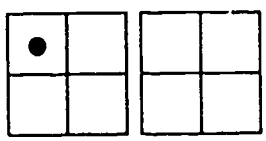
T: This is 20.



T: And what do you think this number is? (40)

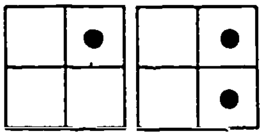


T: What is this number? (80)



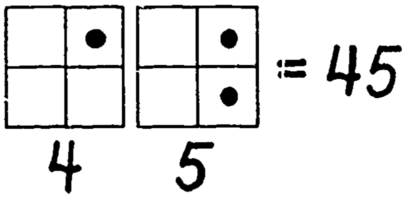
To emphasize the doubling pattern, move one checker to represent 1, 2, 4, 8, and 10, 20, 40, 80. Then, in the order given, let students identify 1, 10, 2, 20, 4, 40, 8, 80 on the Minicomputer. Tell students that we call the boards the "ones' board" and the "tens' board".

Put this configuration on the Minicomputer.



T: What number is this? (45)

Write "45", both below (or above) and beside the Minicomputer. Mention that 45 has four tens and five ones.



In a similar manner, put 2, 3, 17, 88, 34, 68, 70 on the Minicomputer and ask students to identify each number.

Invite students to put 42, 90, 14, 53, 77 on the Minicomputer. Occasionally encourage more than one solution.

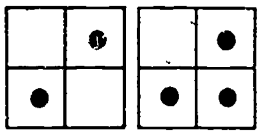
Distribute an individual Minicomputer to each student and direct students to put 28, 44, 81, 30, 59, and 76 on their Minicomputers one at a time. Check a few students' answers for each problem before asking a student to put a solution on the demonstration Minicomputer.

ACTIVITY W6: ADD A CHECKER #1

PREREQUISITE: Activity W5

OBJECTIVE: Students will find all the numbers that can be represented by putting exactly one more checker on the Minicomputer.

Put 67 on the Minicomputer with blue checkers only.



T: What number is this? (67)

Hold up a yellow checker.

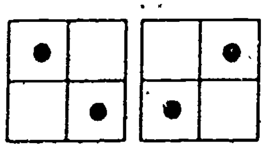
T: What numbers can be put on this Minicomputer with exactly one more checker?

Perhaps a student will place the yellow checker on the 10-square, announce that the new number is 87, and explain that the yellow checker added 20 to the number, and 67 + 20 = 87. Other students may add the values of all the checkers to reach 87.

Remove the yellow checker so that 67 is again on the Minicomputer.

T: Let's find all the numbers that can be put on this Minicomputer with exactly one more checker. (68, 69, 71, 75, 77, 87, 107, and 147)

Repeat this activity as a class, small-group, or individual exercise, with the configuration below. Ask the students to list the numbers that can be put on this Minicomputer with exactly one more checker. (97, 98, 100, 104, 106, 116, 136, 176)



ACTIVITY W7: COMBINATORIAL PROBLEMS #2

PREREQUISITE: Activity W2

OBJECTIVE: Students will find all numbers that can be put on the ones' board with exactly three checkers.

Display one Minicomputer board.

T: What numbers can be put on this Minicomputer with exactly three checkers?

As numbers are suggested, ask students to put them on the Minicomputer.

T: What is the largest possible number? (24)
What is the smallest possible number? (3)
Is it possible to put all the numbers between 3 and 24 on the Minicomputer with exactly three checkers?

Let students work on this problem as a class, in small groups, or individually. The numbers 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 20, and 24 are possible. Encourage students to explain why some numbers between 14 and 24 are impossible.

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ACTIVITY W8: MINICOMPUTER NIM #2

PREREQUISITE: Activities W4 and W5

OBJECTIVE: Play Minicomputer Nim with two boards.

Divide the class into two teams and play Minicomputer Nim as described in Activity W4, using two Minicomputer boards and a goal of 100. Play as many games as time allows.

Minicomputer Nim is dull if students play too cautiously by adding checkers only to the 1-square, 2-square, and 4-square. If this happens in your class, play a few games against the whole class. On your turns, include several plays on the tens' board. Even if you occasionally lose, students will realize that bold moves are acceptable and can lead to a win.

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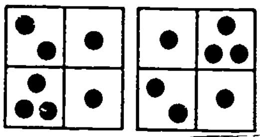
ACTIVITY W9: REMOVE A CHECKER #1

PREREQUISITE: Activities W5 and W6

OBJECTIVE: Given a specific configuration on the Minicomputer, students will determine all numbers that can be represented by removing exactly one checker from the board.

Display this configuration and ask students to calculate the number. (295)



Emphatically remove one checker from the 4-square.

T: What number is on the Minicomputer now? (291) How do you know? (It is 4 less, and 295 - 4 = 291.)

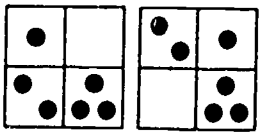
Replace the checker that was removed so that 295 is again on the Minicomputer. Remove one checker from the 40-square.

T: What number is on the Minicomputer? (255)
How do you know? (295 - 40 = 255)

Replace the checker.

T: Let's list all the numbers that can be put on this Minicomputer by removing exactly one of these checkers. (215, 255, 275, 285, 287, 291, 293, and 294)

Repeat this exercise as a full-class, small-group, or individual problem, with the following configuration. Ask the students to list all the numbers that can be put on this Minicomputer by removing exactly one checker. (93, 153, 163, 165, 169, 172)



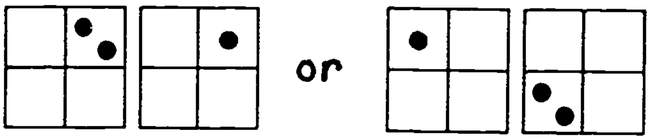
ACTIVITY W10: MISCELLANEOUS PROBLEMS #2

PREREQUISITE: Activities W3 and W5

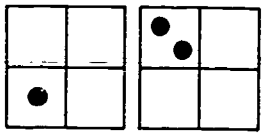
OBJECTIVE: Students will put numbers on the Minicomputer with a specific number of checkers.

Present problems similar to the problems in Activity W3, but use two Minicomputer boards. Create problems according to the ability level of the students. For example,

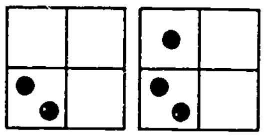
T: Put 84 on the Minicomputer with three checkers.



Put on 36 with three checkers.



Put on 52 with five checkers. (There are several solutions.)



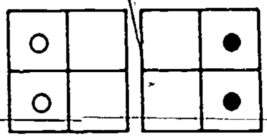
Divide the class into groups of two, three, or four students or let students work individually. Give each group an individual Minicomputer set. Create a series of problems similar to the ones shown here and write them on the board or design a worksheet using the Minicomputers on page 93.

ACTIVITY W11: TUG OF WAR #1

PREREQUISITE: Activity W5

OBJECTIVE: Introduce the game Minicomputer Tug of War.

Put this configuration on the Minicomputer with yellow checkers (shown here by ○) and blue checkers (shown here by ●).



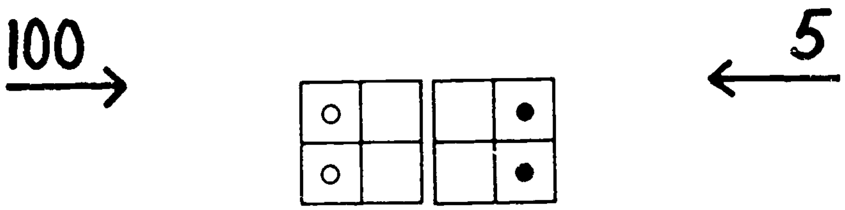
T: We are going to play a game called "Minicomputer Tug of War".

Divide the class into two teams--a Yellow Team and a Blue Team.

T: The Yellow Team may move only yellow checkers. The starting number for the Yellow Team is shown on the Minicomputer with yellow checkers. What number is it? (100)

Write "100" on the board in yellow above and to the left of the Minicomputer.

The Blue Team may move only the blue checkers. What is the starting number for the Blue Team? (5) Write "5" on the board in blue above and to the right of the Minicomputer.



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T: Teams take turns during the game. Players on the Yellow Team move one yellow checker to make the Yellow Team's number smaller. Players on the Blue Team move one blue checker to make the Blue Team's number larger. Right now the Yellow Team's number is larger than the Blue Team's number. As we play, the Yellow Team's number gets smaller and the Blue Team's number gets larger. Eventually the two teams's numbers will pass or tie. The team that passes or ties the other team loses the game.

In other words, the Yellow Team loses by making the yellow number less than or equal to the blue number. Similarly, the Blue Team loses by making the blue number greater than or equal to the yellow number.

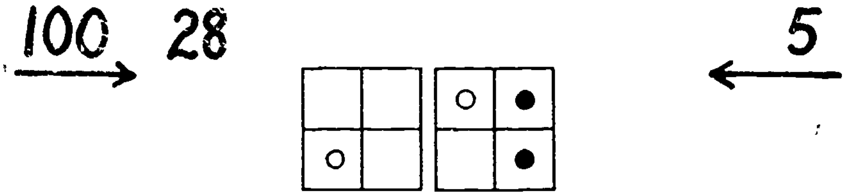
Begin playing the game. Either require complete silence or allow team members to confer, but not to call out instructions to the player moving a checker on the Minicomputer.

Let students volunteer to make moves during the first few games; later, ask them to play in order. This will speed up the early games and allow students to gain familiarity with the game before being required to play.

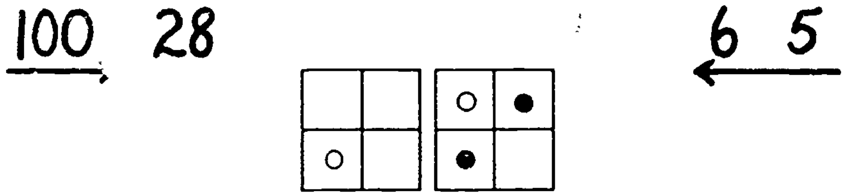
After each play, record the new yellow (blue) number as shown on the next page. As the game progresses, note that the teams's numbers get closer to each other.

An unusually short game is illustrated below.

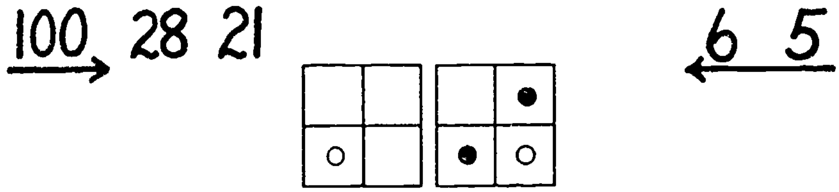
First Play:



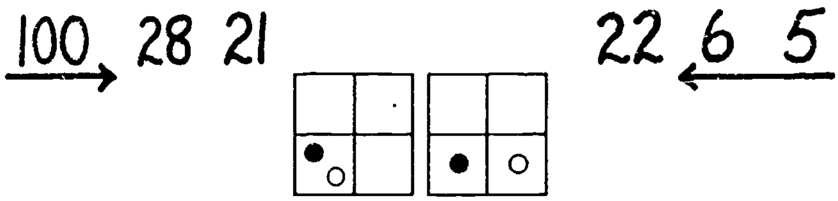
Second play:



Third Play:



Fourth Play:



The Blue Team loses, since 22 is not less than 21.

ACTIVITY W12: REVIEW #1

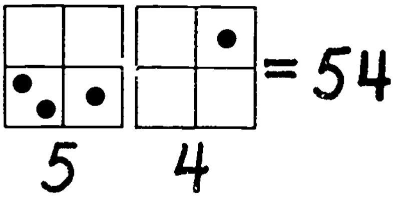
PREREQUISITE: Activity W5

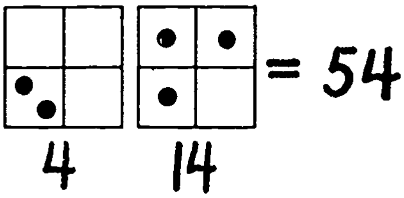
OBJECTIVE: Students will review using two Minicomputer boards to represent numbers.

Display two Minicomputer boards. As a review, put several numbers on the Minicomputer and let students identify them.

Distribute individual Minicomputers. Ask them to put some numbers (e.g. 42, 68, 79) one at a time on their Minicomputers. Check several students' solutions to each problem before letting a student show a solution at the board.

Use two pairs of Minicomputer boards to show these two configurations to the class. Ask students to identify each number (54) and record the numbers as shown below.





Compare the two configurations within a discussion of place value. Note that for the configuration on the left, the "5" and "4" written beneath the boards denote 54, but on the right, the "4" and "14" do not denote 54. Conclude that this type of situation occurs whenever the value of one board is 10 or more.

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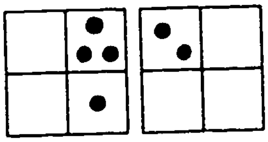
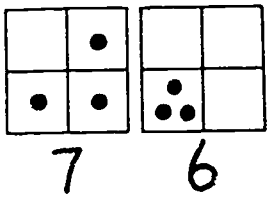
Put several numbers on two Minicomputer boards and let students identify them. As indicated below, distinguish between problems that require addition and those that do not. For example,


$= 130 + 16 = 146$



$= 76$

If many of your students have difficulty in identifying and representing numbers on two Minicomputers, repeat selected exercises from Activities W5 through W9. Do not proceed to Activity W13.

ACTIVITY W13: INTRODUCTION TO THE MINICOMPUTER #3

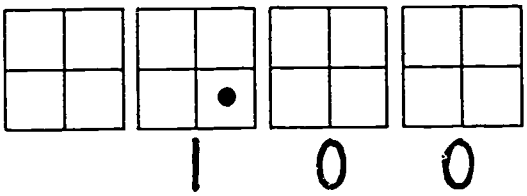
PREREQUISITE: Activity W12

OBJECTIVE: Students will identify and represent numbers on four boards of the Minicomputer.

Display four Minicomputer boards and put a checker on the 100-square.

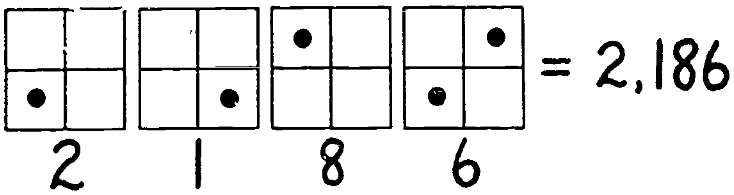
T: What number do you think this is? (100) Why?

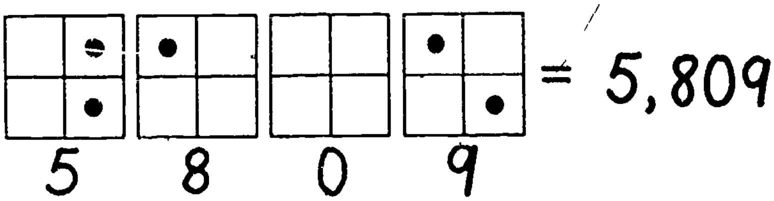
Students might either notice the pattern of the white squares of the Minicomputer--1, 10, 100--or assign a digit to each Minicomputer board.



Use one checker to show 200, 400, 800, 1000, 2000, 4000, and 8000 in order, letting students identify each number. Emphasize both the doubling patterns (100, 200, 400, 800 and 1000, 2000, 4000, 8000) and the x10 patterns (1, 10, 100, 1000 and 2, 20, 200, 2000).

Put several more numbers on the Minicomputer and let students identify them. Occasionally assign a digit to each Minicomputer board and emphasize place value. For example,





93

As a change of pace, ask a student to name any four-digit number. Ask another student to put that number on the Minicomputer. Repeat this activity with other numbers.

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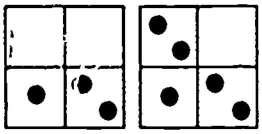
ACTIVITY W14: MINICOMPUTER NIM #3

PREREQUISITE: Activity W8

OBJECTIVE: Students will find one-move and two-move wins in Nim and then play the game in small groups.

Display this configuration on two Minicomputer boards and set a goal of 100.



Goal = 100

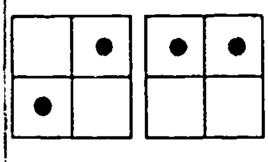
Distribute individual Minicomputer sets and tell the students to put on this exact configuration.

T: Pretend it is your turn in Minicomputer Nim. There is a winning move. Can you find it?

Check several students' answers privately.

T: What is the winning move? (Put a checker on the 40-square.) Why? (60 is on the Minicomputer. 60 + 40 = 100.)

Put this configuration on the Minicomputer.



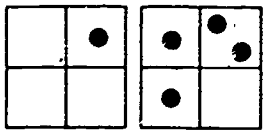
Goal = 100

T: This time there is no winning move, but you can win in two moves. Can you put 100 on the Minicomputer with exactly two more checkers? The other checkers cannot be moved.

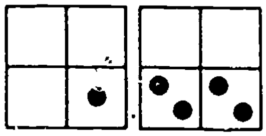
After several students have shown you their answers, let a student announce the solution. (Place one checker on the 20-square and one checker on the 8-square.)

Present these two problems
in a similar manner.

T: Put 100 on this Minicomputer
by adding exactly two checkers.
(Put one checker on the 40-square
and one checker on the 2-square.)



T: Put 100 on this Minicomputer
by adding exactly two checkers.
(Put checkers on the 80-square
and on the 4-square.)



Review the rules for Minicomputer Nim. Divide your class into groups of two, four, or six students that will separate into two teams to play Nim. Let each group set its own goal and decide how many boards to use.

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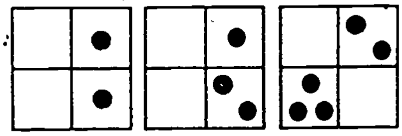
ACTIVITY W15: ADD A CHECKER #2

PREREQUISITE: Activities W6 and W13

OBJECTIVE: Students will determine all numbers that can be formed by putting exactly one more checker on the Minicomputer.

Briefly review representing four-digit numbers on the Minicomputer (see Activity W13). Name some four-digit numbers and ask students to put these numbers on their individual Minicomputers.

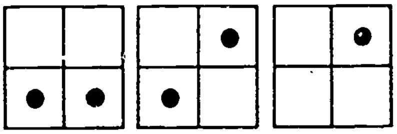
Use three boards to display this configuration and ask students to put it on their individual Minicomputers. Let them calculate the number. (574)



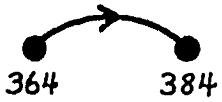
T: What numbers can I put on this Minicomputer with exactly one more checker?

Let students state and explain several solutions as in Activity W6. Then let students work individually or in pairs to find all the solutions. (575, 576, 578, 582, 584, 594, 614, 654, 674, 774, 974, 1374)

Put this configuration on the Minicomputer for identification. (364)

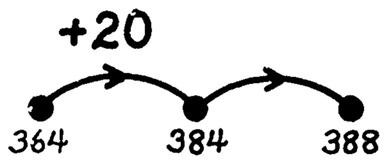


Draw this arrow picture.



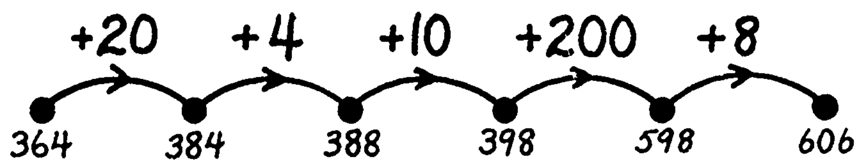
T: The number 364 is on the Minicomputer. Who can put 384 on the Minicomputer by adding exactly one checker? (Put a checker on the 20-square.) How do you know it is 384? (364 + 20 = 384)

Label the arrow "+20" and draw another arrow.



T: Who can reach 388 by adding exactly one more checker? (Put a checker on the 4-square.) How do you know this number is 388? (384 + 4 = 388)

Continue drawing one arrow at a time, asking students to place one more checker each time to show the new number, until this arrow road is completed.



Repeat this exercise with other sequences.

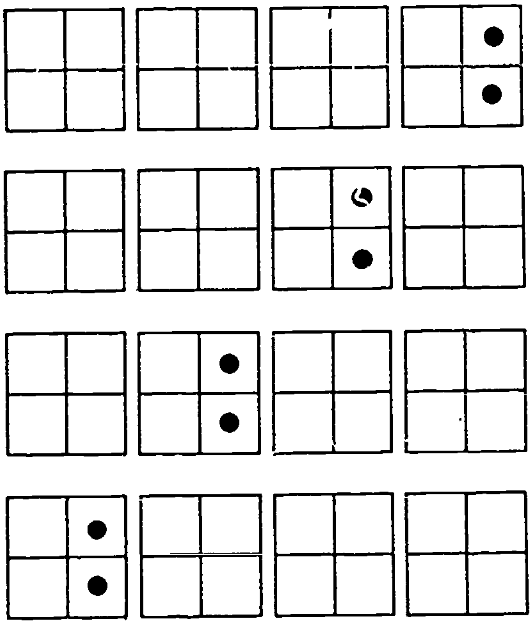
ACTIVITY W16: PATTERNS #1

PREREQUISITE: Activity W13

OBJECTIVE: Students will recognize and use patterns that aid in putting numbers on the Minicomputer.

Review Activity W13 by asking students to identify and represent numbers on four Minicomputer boards.

After the review, put these configurations on the Minicomputer and ask students to identify each number.



5

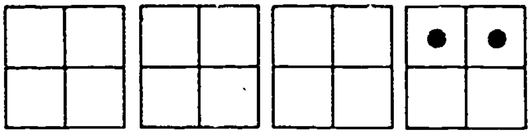
50

500

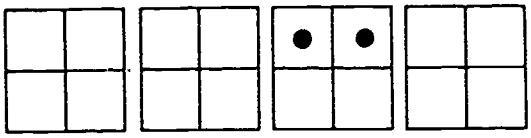
5,000

Similarly, present 9, 90, 900, 9000. Then mix up the order in the patterns. For example, present 600, 6, 6000, 60 and 300, 30, 3000, 3. Also present 24, 240, 2400 and 84, 840, 8400.

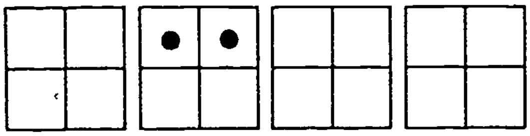
Present this series of configurations:



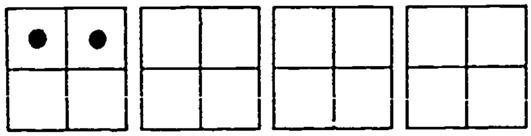
12



120

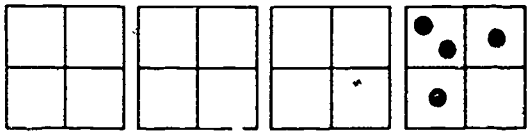


1,200



12,000

Put 22, 220, 2200, and 22,000 on the Minicomputer starting with this configuration.

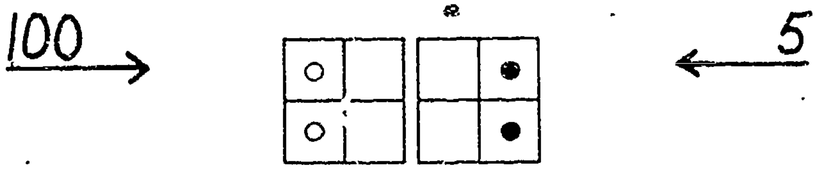


ACTIVITY W17: TUG OF WAR #2

PREREQUISITE: Activity W11

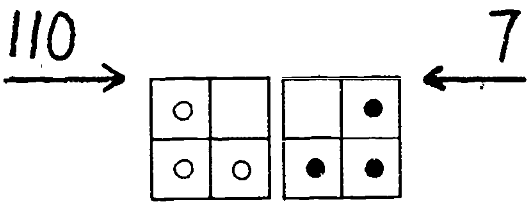
OBJECTIVE: Students will play Tug of War in small groups.

Put this configuration on the Minicomputer and play Tug of War as a class, as described in Activity W11.

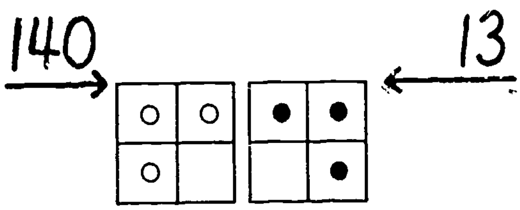


Play one or more games as a class. When most students understand the rules, divide the class into groups of four or six students. Divide each group into a Yellow Team and a Blue Team to play several games of Tug of War. Encourage team members to cooperate.

After a while, suggest that some groups vary their starting configuration. For example,



or

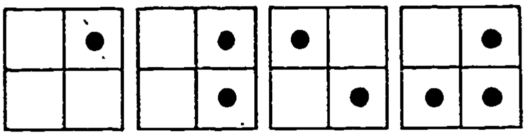


ACTIVITY W18: REVIEW #2

PREREQUISITE: Activity W13

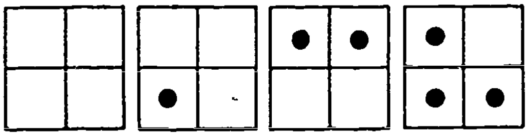
OBJECTIVE: Students will review representing numbers on four Minicomputer boards.

Display four Minicomputer boards and distribute the individual Minicomputer sets. Put several numbers on the Minicomputer and ask students to identify them (students may wish to write down their answers). For example,



= 4597

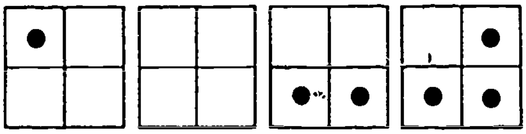
Include some configurations that require mental computation rather than simply aligning the digits. For example,



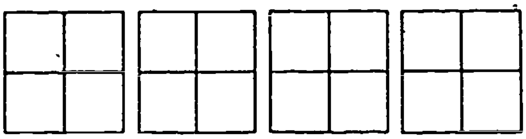
= 200 + 120 + 11 = 331

Name some numbers, one at a time, and ask students to put them on their Minicomputers--for example, 8546, 6052, and 9874.

Alternative: Use the Minicomputers on page 93 to make a worksheet. Ask students to both identify numbers and to put numbers on the Minicomputer. For example,



= ______



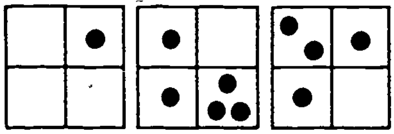
= 6035

ACTIVITY W19: REMOVE A CHECKER #2

PREREQUISITES: Activities W9 and W15

OBJECTIVE: Students will form new numbers by removing exactly one checker from the Minicomputer.

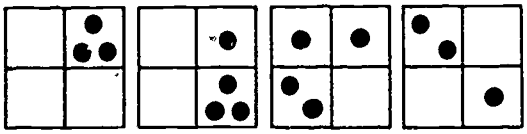
Put this configuration on the Minicomputer for identification. (552)



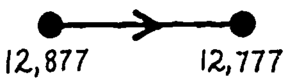
T: What numbers can I put on this Minicomputer by removing exactly one checker?

A student might represent 532 by taking a checker from the 20-square (552 - 20 = 532). Replace that checker and ask students to find all the numbers that can be put on the Minicomputer by removing one checker from this configuration. (152, 472, 532, 542, 544, 548, 550) Conduct this as a class, small-group, or individual activity.

Put this configuration on the Minicomputer for identification. (12,877)



Draw this arrow picture.



T: How can you put 12,777 on the Minicomputer by removing exactly one checker? (Remove a checker from the 100-square.) Explain. (12,877 - 100 = 12,777)

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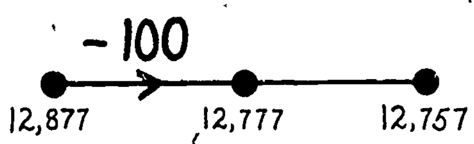
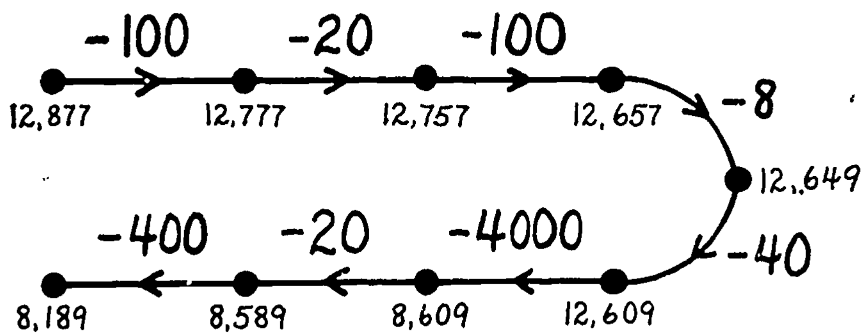
Label the arrow and draw a second arrow.



Continue as in Activity W15, but tell students to remove one checker for each move until this arrow road is completed.



Repeat this exercise with other sequences.

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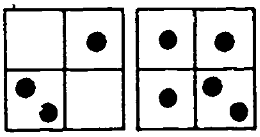
ACTIVITY W20: MINICOMPUTER NIM #4

PREREQUISITE: Activity W14

OBJECTIVE: Students will analyze and play Minicomputer Nim in small groups.

Divide the class into groups of two, four, or six students and give each group an individual Minicomputer. Display this configuration.



T: Let's pretend we are playing Minicomputer Nim and the goal is 120. What number is on the Minicomputer? (96) Can you win in one move? (No) Why not?

You can win in two moves. Put this number on your Minicomputer and try to represent 120 by putting exactly two more checkers on the Minicomputer. (Add a checker to the 4-square and to the 20-square.)

Create and present more problems of this type. Either write the problems on the board or make a worksheet using the Minicomputers on page 93.

Divide each group into two teams. Let them select a goal and play several games of Minicomputer Nim.

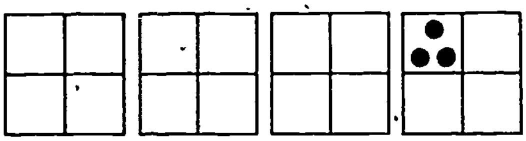
Note: This is the last time Minicomputer Nim appears in this strand. Continue to present similar problems and to play the game whenever convenient if the students are interested.

ACTIVITY W21: PATTERNS #2

PREREQUISITE: Activities W16 and W18

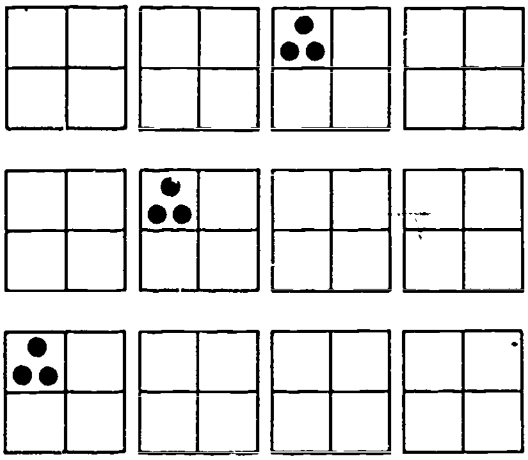
OBJECTIVE: Students will solve multiplication problems by using patterns on the Minicomputer.

Put this configuration on four Minicomputer boards.



T: What number is this? (24) How do you know? (8 + 8 + 8 = 24 or 3 x 8 = 24)

Present these configurations and ask students to identify each number. Encourage them to discuss the pattern.



Conduct a similar discussion with 4 checkers on the purple square (16, 160, 1600, 16000) and 5 checkers on the brown square (40, 400, 4000, 40000).

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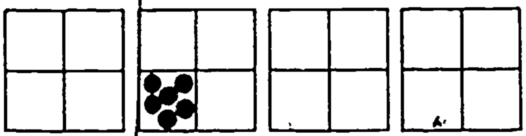
Put this configuration on the Minicomputer.



T: What number is this? (6 x 200 or 1200) How do you know?

Accept students' explanations--for example, "6 x 2 = 12, and 6 x 20 = 120, so 6 x 200 = 1200" or "6 x 2 = 12 and add two zeroes, so 6 x 200 = 1200".

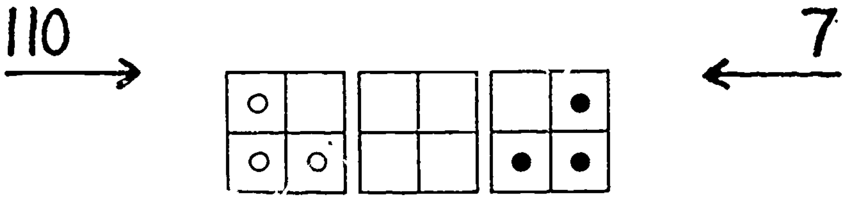
Present similar problems on the Minicomputer. For example, 4 x 80, 3 x 400, 9 x 2000.

ACTIVITY W22: TUG OF WAR #3

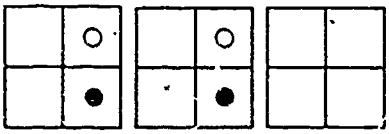
PREREQUISITE: Activities W17 and W18

OBJECTIVE: Students will play Minicomputer Tug of War and find winning moves in various game situations.

Review the rules for Minicomputer Tug of War with the class (see Activities W11 and W17). Divide the class into two teams and play one game as a class using this starting configuration.



Divide the class into groups of two, four, or six students. Tell them that before they play Tug of War, they must solve a few problems. The members of each group can work together. Display this configuration on the demonstration Minicomputers. (These problems may be put on a worksheet.)



T: This situation arose when two teams were playing Tug of War. What is the Yellow number? (440) What is the Blue number? (110)

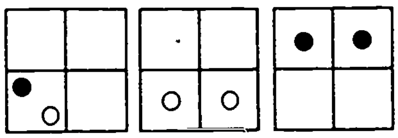
Write the numbers on the board, underlining "440".

T: It is Yellow Team's turn and the Yellow number must be made smaller. The Yellow Team can win on this move. Try to find the winning move for the Yellow Team.

After several groups of students have found the winning move, let a student state the solution. (Move the yellow checker from the 400-square to the 80-square.) The new Yellow number is 120 and the Blue Team must lose.

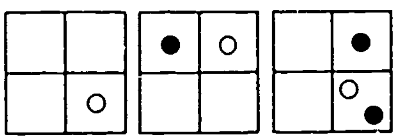
Present these problems in a similar manner.

Yellow to move and win.



Solution: The numbers are 230 and 212. Yellow moves from the 20-square to the 4-square.

Blue to move and win.



Solution: The numbers are 141 and 85. Blue moves from the 1-square or the 4-square to the 40-square.

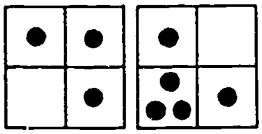
Let the students play Tug of War in small groups. Let each group decide on the number of boards and number of checkers to use in their starting configuration.

ACTIVITY W23: MOVING A CHECKER #1

PREREQUISITE: Activities W15 and W19

OBJECTIVE: Given a configuration on the Minicomputer, students will determine the new number that is formed when one checker is moved from one square to another on the Minicomputer.

Put this configuration on the Minicomputer for identification. (145)



T: We know what happens when a checker is removed from or is put on the Minicomputer. What happens when a checker is moved from one square to another?

Move one checker from the 2-square to the 8-square.

T: What number is on the Minicomputer? (151) How do you know?

Accept all appropriate explanations. Encourage the explanations given below.

- A checker was taken from the 2-square, which makes the number $145 - 2 = 143$. Then it was put on the 8-square, so the new number is $143 + 8 = 151$.

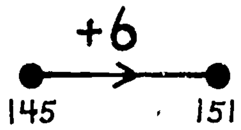
- You moved a checker from the 2-square to the 8-square, so the new number is 6 larger and $145 + 6 = 151$.

Note: The following may help clarify the explanations above.

- If you lend \$2 and are repaid \$8, how much money have you gained?

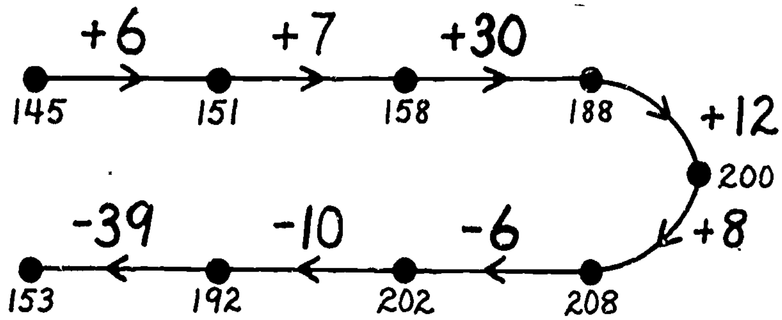
- If a snack stand starts the day with \$2 and ends with \$8 in the cash register, how much money was taken in?

Record this move in an arrow picture.



Continue in a similar manner as indicated below. Move one checker at a time and ask for the new number. Encourage students to use the explanations given above. Occasionally add the values of the checkers to confirm that these methods are accurate. Record each move in the arrow picture.

- Move a checker from the 1-square to the 8-square.
- Move a checker from the 10-square to the 40-square.
- Move a checker from the 8-square to the 20-square.
- Move a checker from the 2-square to the 10-square.
- Move a checker from the 8-square to the 2-square.
  (Note: The new number is smaller.)
- Move a checker from the 20-square to the 10-square.
- Move a checker from the 40-square to the 1-square.



Continue in a similar manner.

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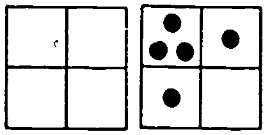
ACTIVITY W24: MISCELLANEOUS PROBLEMS #3

PREREQUISITE: Activity W10

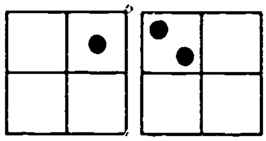
OBJECTIVE: Students will put a number on the Minicomputer with a specified number of checkers.

Present problems similar to those in Activities W3 and W10, according to the level of ability of your students. For example:

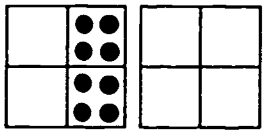
T: Put 30 on the Minicomputer with five checkers. Many solutions are possible.



T: Put on 56 with three checkers.



T: Put on 200 with eight checkers. Many solutions are possible.



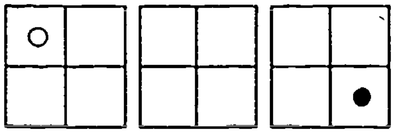
Continue with class, small-group, or individual activities or use the Minicomputers on page 93 to create a worksheet.

ACTIVITY W25: TUG OF WAR #4

PREREQUISITE: Activity W22

OBJECTIVE: Students will conclude that a simple game of Tug of War is trivial.

Review the rules for Tug of War. Divide the class into groups of two, four, or six students. Display this configuration on the Minicomputer.



T: Each group will play Tug of War with this starting configuration.

Most groups will quickly realize that the above game is trivial: the first player can win immediately. If Yellow is first, the Yellow Team can move its checker from the 800-square to the 2-square. If Blue is first, the Blue Team can win by moving its checker from the 1-square to the 400-square. Once a group realizes this, encourage them to play with more checkers and to change the number of boards as they did in Activity W22.

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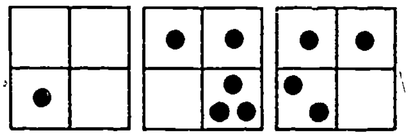
ACTIVITY W26: MOVING A CHECKER #2

PREREQUISITE: Activity W23

OBJECTIVE: Students will put numbers on the Minicomputer by moving a checker from one square to another square.

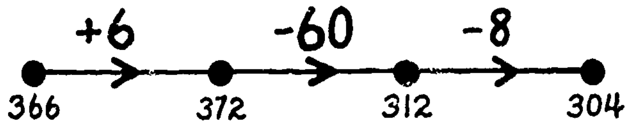
Put this configuration on the Minicomputer for identification. (366)



Create and present problems similar to those in Activity W23. You might start with these moves.

- Move a checker from the 4-square to the 10-square.
- Move a checker from the 80-square to the 20-square.
- Move a checker from the 10-square to the 2-square.

Record each move in an arrow picture.



ERIC

ACTIVITY W27: PATTERNS #3

PREREQUISITE: Activity W21

OBJECTIVE: Students will use place value and patterns to identify numbers on the Minicomputer.

Present patterns similar to those in Activities W16 and W21. At first, present the pattern in its natural sequence. For example, put 6 checkers on the red squares in the sequence 12, 120, 1200, 12000. Then present sequences out of order. For example, put 4 checkers on the brown squares in the sequence 3200, 32, 32000, 320.

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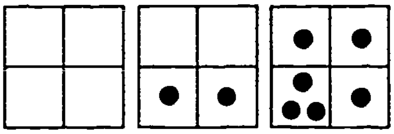
ACTIVITY W28: MINICOMPUTER GOLF #1

PREREQUISITE: Activity W26

OBJECTIVE: Students will play Minicomputer Golf.

Put this configuration on the Minicomputer for identification. (49).



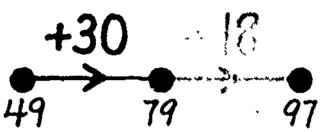
Goal = 200

T: We will play a new game called Minicomputer Golf. Golf is similar to Nim. There will be two teams, a starting number of 49, and a goal of 200. In Nim, each player puts one more checker on the Minicomputer. In Golf, we do not change the number of checkers. Instead, each player moves one checker from one square to another. The teams alternate turns. You are allowed to go over 200, but to win you must reach exactly 200.

Divide the class into a Yellow Team and a Blue Team and invite a player from the Yellow Team to make the first move. Record the results of each move with an arrow picture on the board.

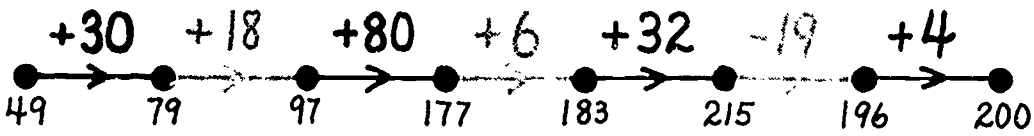
A sample game is recorded here.

The player from the Yellow Team moved a checker from the 10-square to the 40-square. The player for the Blue Team moved a checker from the 2-square to the 20-square.



Alternating yellow and blue chalk provides an effective means for distinguishing between the teams's moves.

The game continues in this manner until the goal of 200 is reached. The following arrow picture describes the entire game. The Yellow Team wins.



After the first game, introduce this rule:

When the number on the Minicomputer is less than the goal, the next player must increase the number. When the number on the Minicomputer is more than the goal, the next player must decrease the number.

Play the game two or three times. Refrain from making comments on the quality of moves. If you spot a potential winning move, do not announce it. Let students enjoy the game as they gradually improve their skills.

If, in playing a game, students delay the game by repeated moves or wild oscillations, institute this rule:

Once the goal is exceeded, each player must form a number on the Minicomputer that, if smaller than the goal, is not smaller than the closest previous approach from below and if larger than the goal is not larger than the closest previous approach from above. For example, in the game above, once 215 is reached, the next player must form a number between 183 and 215.

If this rule leads to a situation where no legal move is possible, declare a tie game.

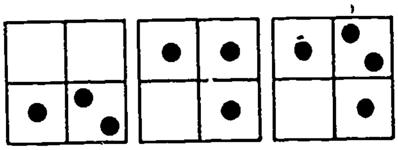
Note: All Activities from W28 to W33 involve Minicomputer Golf. You may wish to intersperse other Minicomputer activities among these activities by creating new problems based on other activities in this section, or by referring to Negative Numbers and Decimals for activities.

ACTIVITY W29: MOVING A CHECKER #3

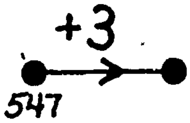
PREREQUISITE: Activities W26 and W28

OBJECTIVE: Students will determine how to increase or decrease a number on the Minicomputer by a specified amount.

Put this configuration on the Minicomputer for identification. (547)



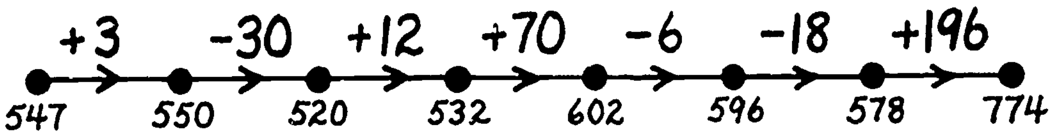
Draw this arrow picture.



T: The number 547 is on the Minicomputer. How can one checker be moved to increase the number by 3? (Move a checker from the 1-square to the 4-square.) Explain.

What number is now on the Minicomputer? (550) How do you know? (547 + 3 = 550)

Continue in a similar manner to construct this arrow picture.

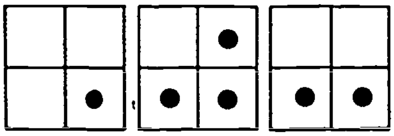


ACTIVITY W30: MINICOMPUTER GOLF #2

PREREQUISITE: Activity W28

OBJECTIVE: Students will develop strategies for playing Minicomputer Golf.

Review the rules of Golf (see Activity W28). Divide the class into two teams and play two or three games of Golf. Vary the starting number and/or goal if you wish. For example,



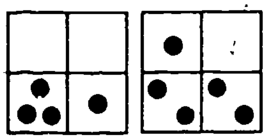
Goal = 500

ACTIVITY W31: MOVING A CHECKER #4

PREREQUISITE: Activities W29 and W30

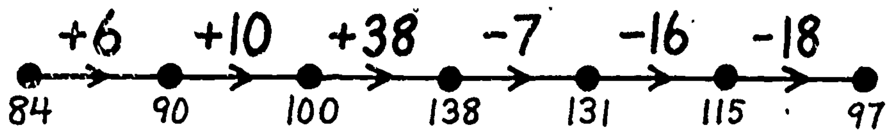
OBJECTIVE: Students will solve problems based on the game of Minicomputer Golf.

Put this configuration on the Minicomputer and ask students to identify the number. (84)



T: The number on the Minicomputer is 84. Who can make it 90 by moving exactly one checker from one square to another square? (Move a checker from the 2-square to the 8-square.) Explain. (Since 90 is 6 more than 84, we need to make the number 6 more.)

Continue in a similar manner to construct this the arrow picture. Draw and discuss one arrow at a time.



ACTIVITY W32: MINICOMPUTER GOLF #3

PREREQUISITE: Activity W30

OBJECTIVE: Students will improve their strategies in playing Minicomputer Golf.

Do one of these two activities:

- Divide the class into groups of four or six students. Let each group divide into two teams and play Golf competitively. Either establish the starting configuration and goal for every group, or let each group determine its own.

- Divide the class into groups of three or four students. At the board, give a starting configuration and goal. Each group must try to achieve the goal in as few moves as possible. After a few minutes let groups demonstrate solutions to the class.

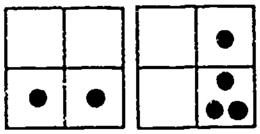
ACTIVITY W33: MINICOMPUTER GOLF PROBLEMS

PREREQUISITE: Activities W31 and W32

OBJECTIVE: Students will solve problems involving situations that could arise in Minicomputer Golf.

This may be a class, small group, or individual activity.

Display this configuration for identification. (37)



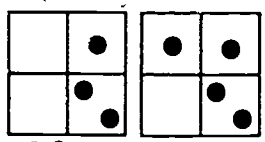
T: Put 51 on the Minicomputer by moving exactly two checkers.

There are five solutions. Move checkers from the

- 10-square to 20-square and 4-square to 8-square;
- 4-square to 20-square and 10-square to 8-square;
- 1-square to 8-square and 1-square to 8-square;
- 20-square to 40-square and 10-square to 4-square;
- 20-square to 4-square and 10-square to 40-square.

Encourage students to find several solutions.

Then repeat the exercise with another configuration. (74)



T: Put 62 on the Minicomputer by moving exactly two checkers.

There are four solutions. Move checkers from the

- 4-square to 1-square and 10-square to 1-square;
- 10-square to 4-square and 10-square to 4-square;
- 10-square to 2-square and 8-square to 4-square;
- 10-square to 4-square and 8-square to 2-square.

To create similar problems, set up a configuration, move two checkers, and calculate the total amount of increase or decrease. Your moves provide one solution. Do not worry about finding more solutions; let the students find them!

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INTRODUCTION TO NEGATIVE NUMBERS ON THE MINICOMPUTER

The story in Activities N1, N2, and N3 provides an introduction to representing negative numbers on the Minicomputer. The story and subsequent activities in this section are meant to enhance a traditional approach to negative numbers. For primary grades, you might use these activities as an introduction to negative numbers. For older students, these activities provide a non-standard, attractive reinforcement. We stress that this approach is meant to supplement, not replace, other classroom work on negative numbers.

ACTIVITY N1: NEGATONS AND POSONS #1

PREREQUISITE: None

OBJECTIVE: Students will learn to add integers using a model based on a science fiction story.

Tell the class the following story.

T: On a planet far away, two countries occupy a large island continent. The peoples call themselves the Negatons and the Posons. A great wall separates the two countries and there is no communication between the countries. In fact, the wall was built so long ago that no one now can remember why it was built. Everyone fears the wall; parents and teachers on both sides tell children never to venture near the wall.

Every year a few children disappear from each country; almost always the missing children were last seen near the wall. The Posons always blame the Negatons and the Negatons always blame the Posons. But no one knows what really happens.

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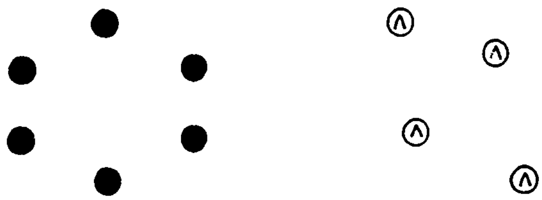
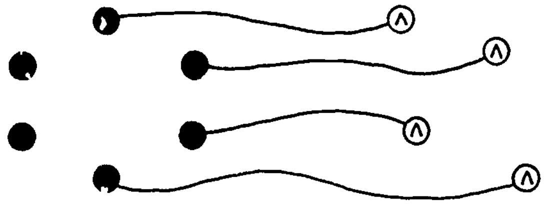
T: Astron, Poi, and Nona, three Poson children, decide to explore the wall one night. They sneak to the wall, climb over, and suddenly are face to face with 2 Negaton children. Afraid, all are ready to run. But Poi and Nona reach out to shake hands with the Negatons. The 2 Negatons smile, step forward, shake hands with Poi and Nona. Poof! All 4 disappear. Astron is alone! Astron climbs back over the wall and runs home to tell the story. The Poson elders are irate. But instead of immediately attacking, they make contact with the Negatons by radio and propose a summit conference. Each country sends 5 leaders to the conference. But as they meet and shake hands, they all disappear!

Both sides suspect trickery and decide to attack. All of the battles are among spaceships. In the first battle, the Posons send 6 spaceships. Their insignia is a large black dot. The Negatons send 4 spaceships. Their insignia is different.



T: Just as pairs of people disappear, pairs of spaceships disappear.



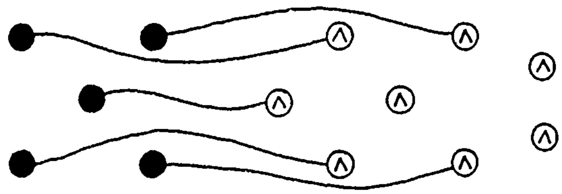
T: How many ships remain? (2) Whose ships? (Posons) The generals record results of the battle in this way:

$$6 + \hat{4} = 2$$

Note: $6 + \hat{4} = 2$ is read, "Six Poson spaceships meet 4 Negaton spaceships, and 2 Poson spaceships survive the battle".

Present more battles in a similar manner. For example, 5 Poson ships meet 8 Negaton ships. What is the result? Encourage drawing pictures as necessary and useful to assist in the computation.

$$5 + \hat{8} = \hat{3}$$



Let students solve problems similar to the following examples, drawing pictures as necessary.

$$9 + \hat{6} = (3)$$

$$\hat{7} + 2 = (\hat{5})$$

$$50 + \widehat{50} = (0)$$

$$70 + \widehat{90} = (\widehat{20})$$

T: Sometimes the Posons send reinforcements. If they send 3 ships to join a fleet of 8 ships they can record it like this. Of course, no ships disappear.

$$8 + 3 = 11$$

T: The Posons also can send reinforcements.

$$\hat{7} + \hat{6} = \widehat{13}$$

Create and present a variety of problems involving Poson ships and Negaton ships. Continually remind students of the story line.

ACTIVITY N2: NEGATONS AND POSONS #2

PREREQUISITE: Activity N1

OBJECTIVE: Students will add integers.

Review the Poson-Negaton story from Activity N1. Ask students to solve problems involving Posons and Negatons. Encourage them to use pictures as in Activity N1 when necessary. For example, present these problems:

$$\hat{9} + 7 = \qquad \hat{3} + 16 =$$

$$\hat{4} + \hat{5} = \qquad 11 + 7 =$$

Continue with another episode of the story.

T: The Posons set up a large base with 17 spaceships. Each day the Negatons send 5 ships to attack the base. How many Poson ships survive the first day? (12: $17 + \hat{5} = 12$) We will use arrows to record the results of the battles.

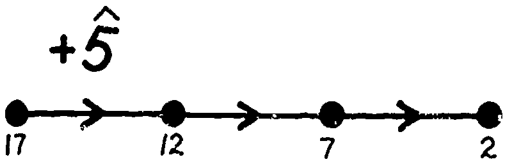
$$17 \xrightarrow{+\hat{5}} 12$$

Note: This arrow picture is read "Seventeen Poson spaceships are attacked by 5 Negaton spaceships and 12 Poson ships survive".

T: How many ships survive the second attack? (7)

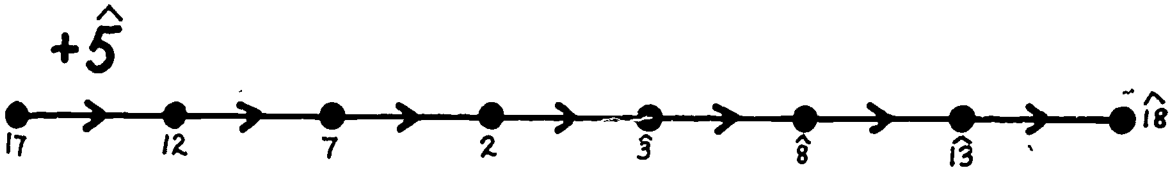
$$17 \xrightarrow{+\hat{5}} 12 \longrightarrow 7$$

Continue similarly until these arrows are drawn.



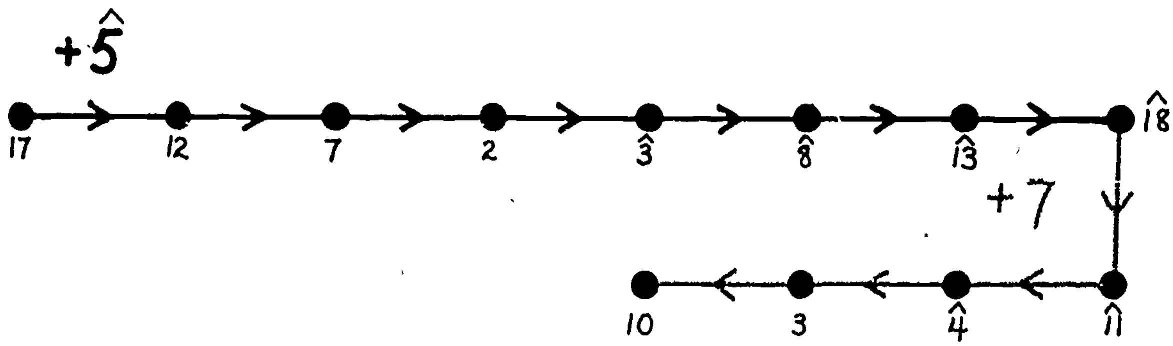
T: What happens when the Negatons use 5 spaceships to attack 2 Poson spaceships? (Three Negaton ships survive: $2 + \hat{5} = \hat{3}$.) The Negatons continue to send 5 ships each day to build a base.

Extend the arrow picture as follows and let students label the dots.



T: Now the Negatons stop sending ships and the Posons begin sending 7 ships to attack the base every day.

Extend the arrow picture and let students label the dots.



Extend the Negaton and Poson story as you wish. Use arrows to record the results of any series of battles.

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ACTIVITY N3: NEGATIVES ON THE MINICOMPUTER

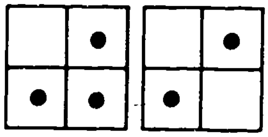
PREREQUISITE: Activities N1 and W12

OBJECTIVE: Students will add integers using the Minicomputer.

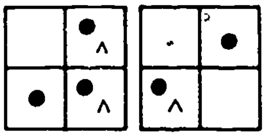
Review the Poson-Negaton story, presenting problems similar to those in Activities N1 and N2.

T: When large numbers of spaceships are involved, the commanders use Minicomputers to calculate the results of the battles. In one battle, 76 Poson spaceships attacked 52 Negaton spaceships. Write "$76 + \hat{52} =$ ___" on the board.

T: First, put 76 on the Minicomputer. These checkers represent the number of Poson ships.

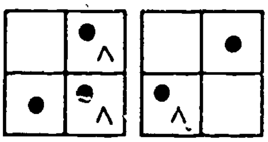
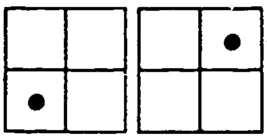


T: Now represent 52 Negaton spaceships, using the special Negaton checkers.

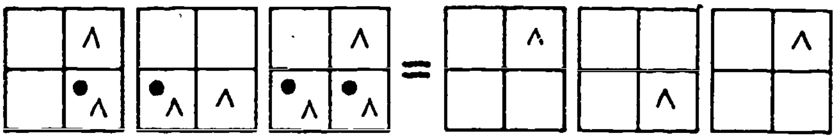


T: How could the commanders use this Minicomputer to calculate the number of survivors of the battle?

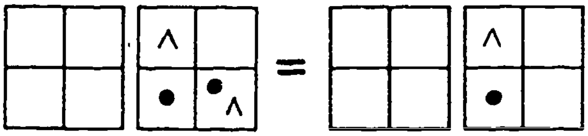
Encourage the idea that a Poson checker and a Negaton checker on the same square can both be removed from the Minicomputer. ($40 + \hat{40} = 0$; $\hat{10} + 10 = 0$; $2 + \hat{2} = 0$; and so on) Thus, in this case,

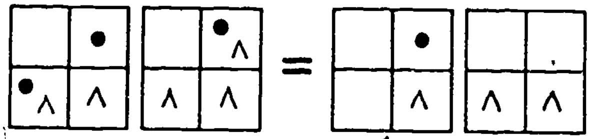
$76 + \hat{52} =$  $=$  $= 24$

Present additional problems in which only Poson checkers or only Negaton checkers remain after all pairs of Poson checkers and Negaton checkers on the same Minicomputer square are removed. For example,

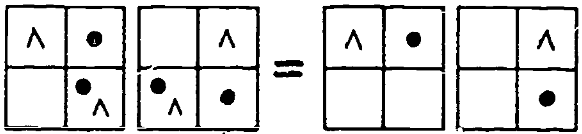
$123 + \widehat{537} =$  $= \widehat{414}$

In most addition problems, even after the removal of all pairs of Poson checkers and Negaton checkers on the same square both types of checkers still remain on the Minicomputer. In this situation, mentally calculate the answer. For example:

$\hat{9} + 3 =$  $= \hat{8} + 2 = \hat{6}$

$64 + \widehat{37} =$  $= 30 + \hat{3} = 27$

$40 + \widehat{10} = 30$ $\quad \hat{3}$

$53 + \widehat{96} =$  $= \widehat{40} + \hat{3} = \widehat{43}$

$\widehat{80} + 40 = \widehat{40}$ $\quad \hat{4} + 1 = \hat{3}$

Present additional, similar problems. Occasionally refer to the Poson-Negaton story. Include a few problems involving only negative numbers or only positive numbers. For example, $\widehat{32} + \hat{7} = \widehat{39}$ and 28 + 15 = 43.

ACTIVITY N4: COMBINATORIAL PROBLEMS #3

PREREQUISITE: Activities N3 and W7

OBJECTIVE: Students will list all numbers that can be put on the ones' board of the Minicomputer with exactly one positive checker and one negative checker.

Review activities N1, N2, and N3 as necessary.

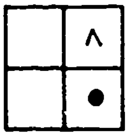
Note: Begin using the terminology "positive" and "negative" and the standard notation ($\hat{3} = -3$) if you wish.

Display one positive checker, one negative checker, and the ones' board of the Minicomputer.

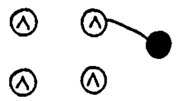
T: What numbers can be put on this Minicomputer using both of these checkers?

After students generate a few solutions, challenge them (individually or as a class) to find all solutions. ($\hat{7}$, $\hat{6}$, $\hat{4}$, $\hat{3}$, $\hat{2}$, $\hat{1}$, 0, 1, 2, 3, 4, 6, 7)

As in the previous activities, remind students of the space battle story and encourage them to draw pictures when necessary to assist in their computation. For example,



$= \hat{4} + 1$



$\hat{4} + 1 = \hat{3}$

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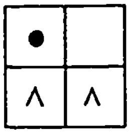
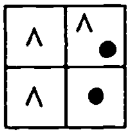
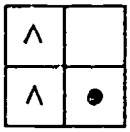
ACTIVITY N5: REVIEW

PREREQUISITE: Activity N4

OBJECTIVE: Students will review putting numbers on the Minicomputer with both positive checkers and negative checkers.

Put a sequence of numbers on the Minicomputer and ask students to identify them. For example,

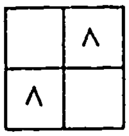
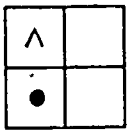
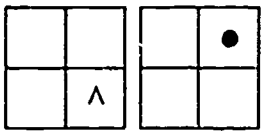
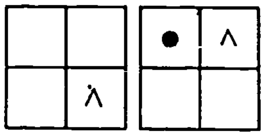
 $= 8 + \hat{3} = 5$

 $=$  $= \hat{10} + 1 = \hat{9}$

Construct similar problems. If desired, ask students to write down their answers or prepare a worksheet for individual work. Students need individual Minicomputers for the following exercise.

T: Put $\hat{6}$ on your Minicomputers. Try to use at least one positive checker and at least one negative checker.

Let several students share their solutions with the class. For example,

 or  or  or 

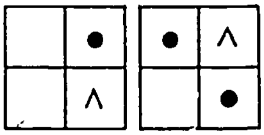
Select other numbers for students to put on their Minicomputers.

ACTIVITY N6: ADD A CHECKER #3

PREREQUISITE: Activities N3 and W15

OBJECTIVE: Students will find numbers that can be put on the Minicomputer with one more negative checker.

Present these problems as you presented Activities W6 and W15. Put this configuration on the Minicomputer.



T: What is this number? (35) How do you know? What numbers can be put on the Minicomputer with exactly one more negative checker? (Suppose that 15 is suggested.) Where should we put the negative checker? (On the 20-square.) How do you know this number is 15?

Several explanations are possible:

- Find the effect of the new checker on the number 35:

  $35 + \hat{20} = 35 - 20 = 15.$
- Calculate each board separately:

  $40 + \hat{30} = 10$, $9 + \hat{4} = 5$, and $10 + 5 = 15$.
- Calculate the positive number and the negative number:

  $49 + \hat{34} = 15.$

Let students put all possible numbers ($\hat{45}$, $\hat{5}$, 15, 25, 27, 31, 33, and 34) that they find on the Minicomputer. Encourage students to use the first of the above methods to explain their results. Observe that putting a negative checker on the Minicomputer results in a smaller number.

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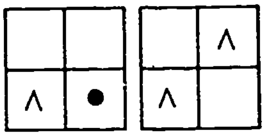
Repeat the activity with this configuration.

 $= \hat{16}$

T: What numbers can be put on the Minicomputer with exactly one more negative checker? ($\hat{96}$, $\hat{56}$, $\hat{36}$, $\hat{26}$, $\hat{24}$, $\hat{20}$, $\hat{18}$, and $\hat{17}$)

Let students solve this problem as a class, in pairs, or individually.

ACTIVITY N7: A GAME WITH NEGATIVE CHECKERS #1

PREREQUISITE: Activity N5

OBJECTIVE: Students put a number on the Minicomputer with a given set of checkers.

Divide the class into two teams. For each team, display two Minicomputer boards, three positive checkers and one negative checker.

T: Each team's goal is to put 13 on its Minicomputer using all four of these checkers. First, a player from Team A places one checker on Team A's Minicomputer. Player B plays second, and cannot copy player A's move.

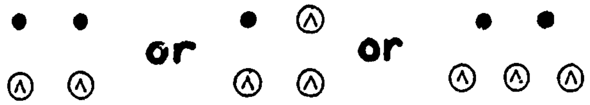
After each round of plays, have the class identify the numbers on each Minicomputer. Then invite another student from Team B to place a second checker on Team B's Minicomputer. Another student from Team A plays second and cannot make the configuration on Team A's Minicomputer the same as the configuration on Team B's Minicomputer.

Continue in a similar manner until all four checkers have been placed on the Minicomputers. Three situations are possible:

- Both teams have 13 in different ways, and the game is a tie;
- Only one team has 13 and is the winner;
- Neither team has 13. In this case, continue the game by letting another player from each team move one checker on the team's Minicomputer from one square to another square. Continue in this manner until at least one team puts 13 on its Minicomputer.

Variations:

- Change the goal. For example, set a goal of 26, 70, $\hat{7}$, or $\widehat{38}$.
- Use a different set of checkers--for example,



- Let students play the game in small groups.

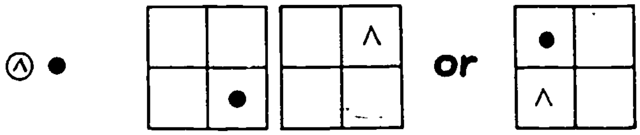
ACTIVITY N8: PROBLEMS

PREREQUISITE: Activities N5 and W10

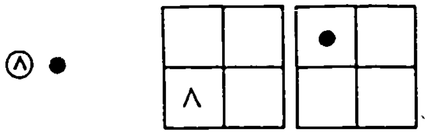
OBJECTIVE: Students will put numbers on the Minicomputer with a specified set of checkers.

Present problems similar to these:

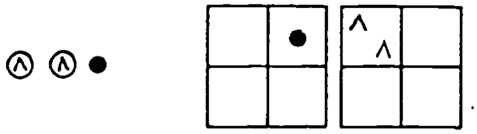
T: Put 6 on the Minicomputer with these checkers:



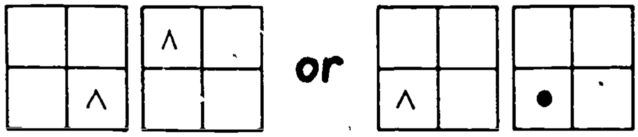
T: Put on $\widehat{12}$ with these checkers:



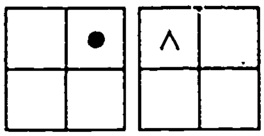
T: Put on 24 with these checkers:



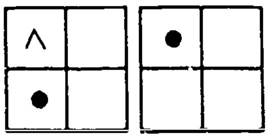
T: Put on $\widehat{18}$ with exactly two checkers (positive or negative).



T: Put on 32 with exactly two checkers.



T: Put on $\widehat{52}$ with exactly three checkers. There are many possible solutions.

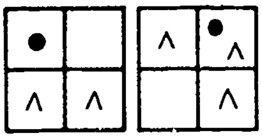


ACTIVITY N9: ADD A CHECKER #4

PREREQUISITE: Activity N6

OBJECTIVE: Students will list numbers that can be formed by putting exactly one more checker on the Minicomputer.

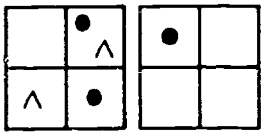
Put 41 on the Minicomputer as shown below and proceed as in Activity N6.



T: What numbers can I put on this Minicomputer with exactly one more positive checker? (42, 43, 45, 49, 51, 61, 81, and 121)

What numbers can I put on this Minicomputer with exactly one more negative checker? ($\widehat{39}$, 1, 21, 31, 33, 37, 39, and 40)

Repeat this activity with this configuration.



$= \hat{2}$

ACTIVITY N10: A GAME WITH NEGATIVE CHECKERS #2

PREREQUISITE: Activity N7

OBJECTIVE: Students will place numbers on the Minicomputer with a given set of checkers.

Play the game as described in Activity N7. Play with two positive and two negative checkers for each team and a goal of 17. Repeat the game several times. Refer to Activity N7 for variations.

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ACTIVITY N11: COMBINATORIAL PROBLEMS #4

PREREQUISITE: Activity N8

OBJECTIVE: Given a set of numbers, students will determine which of them can be put on the Minicomputer with specified checkers.

Display three Minicomputer boards, two positive checkers, and one negative checker. Write these nine numbers on the board.

$$15 \quad \widehat{18} \quad 320 \quad 416 \quad \widehat{52}$$
$$25 \quad 88 \quad \widehat{176} \quad \widehat{33}$$

T: Seven of these nine numbers can be put on the Minicomputer using exactly these three checkers.

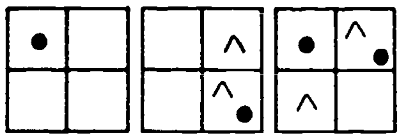
After students put two or three of the numbers on the Minicomputer, complete this activity as a class, in small groups, or individually. (All numbers except 25 and $\widehat{33}$ are possible.)

ACTIVITY N12: REMOVE A CHECKER #3

PREREQUISITE: Activities N9 and W19

OBJECTIVE: Students will determine all numbers that can be put on the Minicomputer by removing exactly one checker.

Ask students to calculate this number. (766)



Remove a positive checker from the 4-square.

T: What number is now on the Minicomputer? (762) Why? (Removing a checker from the 4-square makes the number four smaller. 766 - 4 = 762)

Replace the positive checker on the 4-square. Remove the negative checker from the 4-square.

T: What number is now on the Minicomputer? (770) Why? (Putting on a negative checker makes the number smaller, so removing a negative checker makes the number larger: 766 + 4 = 770.)

Let students find all numbers that can be put on the Minicomputer by removing exactly one checker. (34, 756, 758, 762, 768, 770, 776, and 806) Repeat this exercise with another configuration.

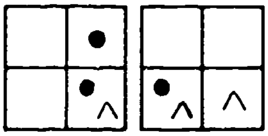
Note: Activities N13 to N16 all involve Minicomputer Golf with negative checkers. Intersperse other activities among these lessons.

ACTIVITY N13: MOVING A CHECKER #5

PREREQUISITE: Activities N9, N12, W30

OBJECTIVE: Students will compare the effects of moving negative and positive checkers on the Minicomputer.

Ask students to identify this number. (39)



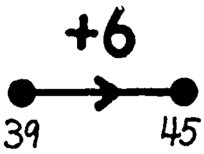
T: The number on the Minicomputer is 39. I will move one checker.

Move a positive checker from the 2-square to the 8-square.

T: What number is on the Minicomputer? (45) How do you know?

See Activity W23 for a discussion of possible explanations.

Record the move with an arrow picture.



Move a negative checker from the 2-square to the 8-square.

T: What number is on the Minicomputer? (39) How do you know?

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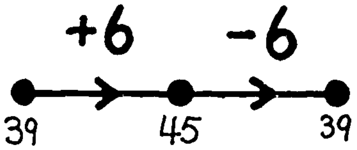
Accept any reasonable explanations, including the following:

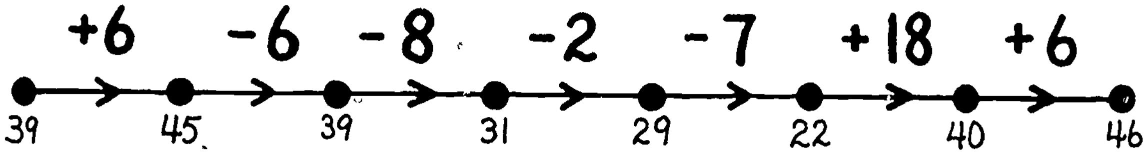
1. Add the values of the checkers on the Minicomputer.
2. The value of the checker changed from $\hat{2}$ to $\hat{8}$, $\hat{8}$ is 6 less than $\hat{2}$ and $45 - 6 = 39$.
3. Moving a negative checker from the 2-square to the 8-square has the opposite effect to moving a positive checker.

Record the move.



Continue in a similar manner as indicated below. Encourage students to use the second or third explanation given above.

- Move a positive checker from the 10-square to the 2-square.
- Move a negative checker from the 8-square to the 10-square.
- Move a negative checker from the 1-square to the 8-square.
- Move a positive checker from the 2-square to the 20-square.
- Move a negative checker from the 10-square to the 4-square.



Continue in a similar manner.

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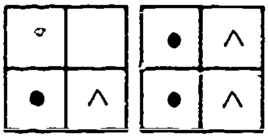
ACTIVITY N14: MINICOMPUTER GOLF #4

PREREQUISITE: Activities N13 and W30

OBJECTIVE: Students will play Minicomputer Golf with positive and negative checkers.

Play golf as described in Activity W28 with the starting configuration (15) and goal shown below. When a negative checker is moved, take care in deciding whether the move increases or decreases the number on the Minicomputer.



GOAL: 60

Play the game as often as time allows. Vary the starting configuration and goal if you wish.

ACTIVITY N15: MINICOMPUTER GOLF #5

PREREQUISITE: Activity N13 and N14

OBJECTIVE: Students will review the numerical effect of moving checkers on a Minicomputer and will play Minicomputer Golf.

Create and present problems similar to those in Activity N13.

Play Minicomputer Golf with positive and negative checkers as described in Activity N14.

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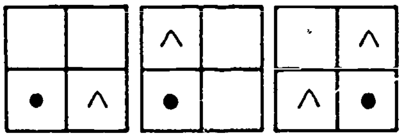
ACTIVITY N16: MOVING A CHECKER #6

PREREQUISITE: Activities N15 and W29

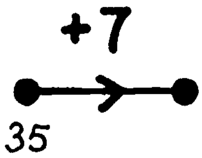
OBJECTIVE: Students will find the move on the Minicomputer which changes the number by a specified amount.

Put this configuration on the Minicomputer.



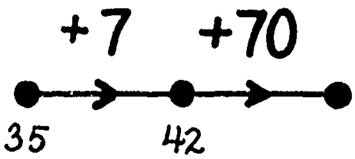
T: What is the number? (35) How do you know?

Begin an arrow road.



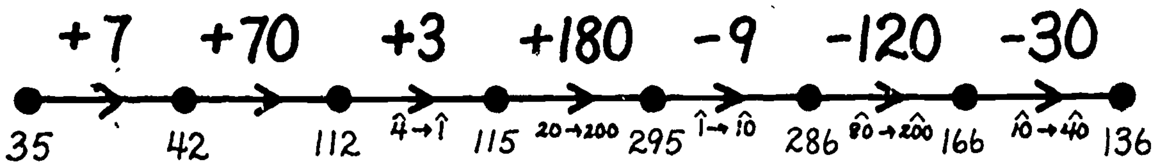
T: The number on the Minicomputer is 35. How can I move one checker to increase the number by 7? (Move a positive checker from the 1-square to the 8-square.) Why? (1 is 7 less than 8.) What number is now on the Minicomputer? (42) How do you know? (35 + 7 = 42)

Label the dot "42" and draw another arrow to propose another problem.



Solution: Move a negative checker from the 80-square to the 10-square. Then 42 + 70 = 112 is on the Minicomputer.

Continue in a similar manner to construct this arrow picture.



Create similar problems suitable for your students.

Note: Play Minicomputer Golf with positive and negative checkers. Present problems similar to those in Activities N13 and N16 as desired. The variations of Golf described in Activity W32 are appropriate to this version of the game also.

INTRODUCTION TO DECIMALS ON THE MINICOMPUTER

The activities in this section require students to be familiar with decimal numbers and to have completed at least Activities W1 through W13. Activities D1 and D2 provide an introduction to representing decimal numbers on the Minicomputer. Since most students' real-world experiences with decimal numbers involve money, these two activities rely heavily on a money model for decimals. Therefore, read 3.6 as "three point six" in the beginning, and interpret it as as three dollars and six dimes or $3.60. The money model provides a foundation on which to develop decimal number concepts and often aids in diagnosing the students' lack of understanding when they give incorrect responses. Once students seem comfortable with the tenths' board and hundredths' board, feel free to introduce additional boards (thousandths', ten-thousandths', etc.). Of course, reliance on the concept of money would then be dropped.

Activities D3 to D11 provide suggestions for adapting activities from the Whole Number Section (W) for use with decimal numbers. The brief descriptions usually include a few sample problems with solutions as a means of emphasizing the changes involved in switching to decimal numbers. Create additional similar problems by referring to the related Whole Number activities. In creating and presenting problems, be sensitive to the abilities of your students: aim to challenge, but not frustrate. The sample problems are usually of medium difficulty and are not necessarily meant to be introductory problems. In preparing a full lesson on decimals, prefer to select and expand problems from several activities rather than to spend a whole lesson on one activity.

The bar shown here in gray separates the ones' board from the tenths' board of the Minicomputer. It is meant to be drawn on the board with white chalk.

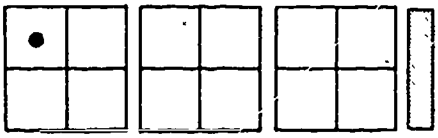
ACTIVITY D1: DECIMALS ON THE MINICOMPUTER #1

PREREQUISITES: Activities W1 through W13

OBJECTIVE: Through patterns, students will learn to represent and identify decimal numbers on the Minicomputer.

Note: Borrowing extra demonstration Minicomputer boards would be useful, though not necessary, for this activity.

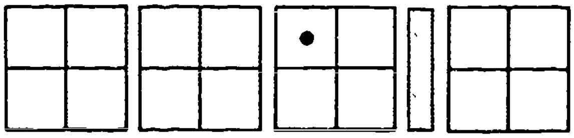
Display three Minicomputer boards. On the chalk board, draw a chalk bar to the right of the Minicomputer. Put a checker on the 800-square.



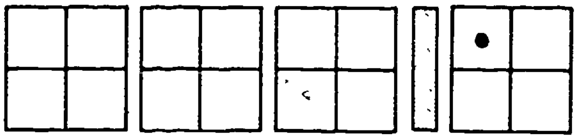
T: What number is this? (800)

Put 80 and 8 on the Minicomputer and ask students to identify them. Add a board to the right of the bar.

T: This is 8.



T: What number is this?



If necessary, tell the students that the number is 0.8 (read 0.8 as "zero point eight").

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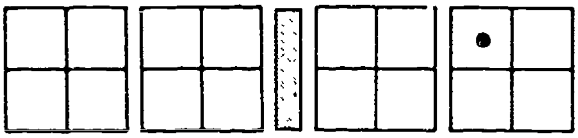
Put this number on the Minicomputer and let students identify it. (0.08)

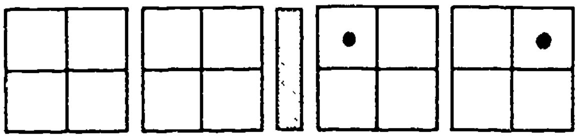


Similarly, let students identify 100, 10, 1, 0.1, 0.01 and 600, 60, 6, 0.6, and 0.06. Discuss the role of the bar as a decimal point. Let students identify 420, 42, 4.2, 0.42. Ask students to put these sequences of numbers on the Minicomputer: 400, 40, 4, 0.4, 0.04 and 180, 18, 1.8, 0.18.

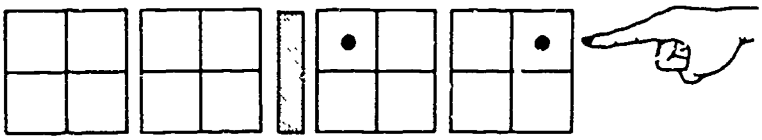
Put numbers such as 3.42, 0.87, 23.08, 23.80, 50.06, 50.60, 5.06, and 5.60 on the Minicomputer, one at a time, and let students identify them. Then ask students to put several numbers that you name on the Minicomputer--for example, 6.48, 0.53, 80.04, 8.04, and 8.40.

Write \$0.84 on the board.

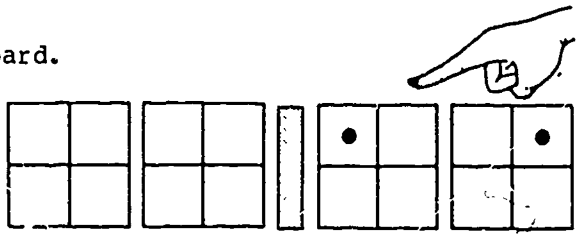
T: How much money is this? (84¢) Put 84¢ on the Minicomputer.



T (pointing to the 4): There are four pennies in 84 cents. So we'll call this (see below) the "pennies' board".

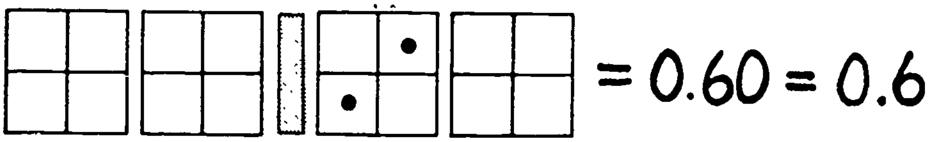


Point to the next board.



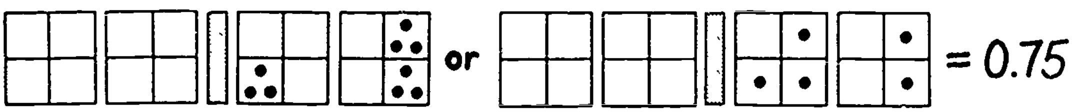
T: What do you think we call this board? ("Dimes' board". There are eight dimes in \$0.84.)

T: Who can put six dimes on the Minicomputer?
Who can write this number as a decimal?

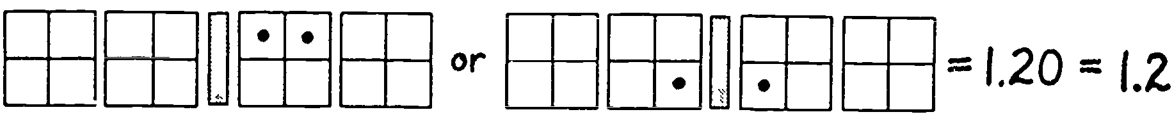


Ask students to show each of the following on the Minicomputer and then to write each amount as a decimal: six pennies, six dollars, and one quarter.

T: Who can put three quarters on the Minicomputer?
Who can write this number as a decimal?



Ask students to put twelve dimes on the Minicomputer and to write the number as a decimal.



Create similar problems if you wish, or use the Minicomputers on page 93 to create worksheets.

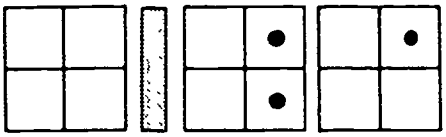
ACTIVITY D2: DECIMALS ON THE MINICOMPUTER #2

PREREQUISITE: Activity D1

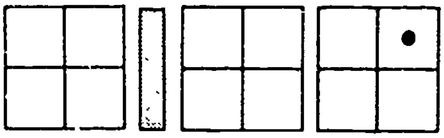
OBJECTIVE: Students will use the Minicomputer and the idea of money to understand that 0.6 = 0.60, not 0.06, and that 16 dimes = \$1.60, not \$0.16.

Spend at least ten minutes reviewing Activity D1. Emphasize the idea of a dimes' board and a pennies' board.

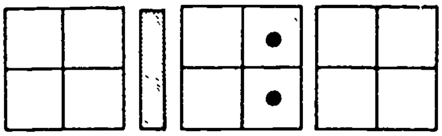
For each of the following configurations, ask students to write the number as a decimal and to state the amount of money represented.


= 0.54

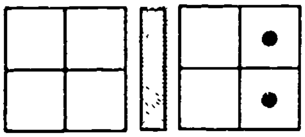
S: Five dimes and four pennies are 54¢.


= 0.04

S: Four pennies.


= 0.50

S: Five dimes equal 50¢.

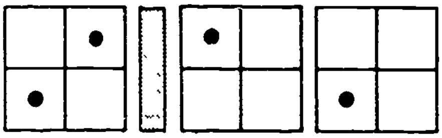

= 0.5

S: Five dimes equal 50¢.

Conclude that 0.5 = 0.50 = 50 cents and, in particular, that 0.05 = 5 cents. Emphasize that 0.5 is not 5 cents.

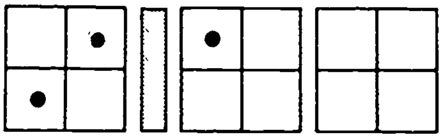
The above conclusion may be difficult for many students to accept and to understand. This difficulty reveals a basic misunderstanding of decimal place value. Do not expect such a weakness to be remedied immediately. Acceptance of this "paradox" will develop slowly through regular exposure to decimal numbers, reliance on the money model, and patience.

Continue this activity with these configurations, asking students to identify each number as an amount of money and to write it as a decimal.



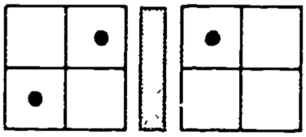
= 6.82

S: $6.82 is six dollars, eight dimes, and two pennies.



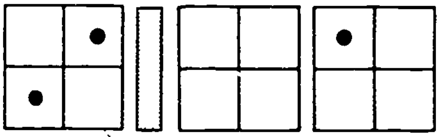
= 6.80

S: $6.80.



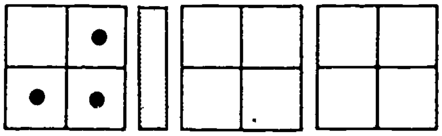
= 6.8 = 6.80

S: Six dollars and eight dimes equal $6.80.



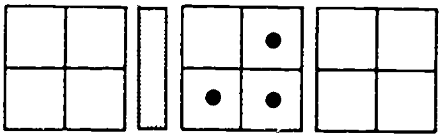
= 6.08

S: $6.08.



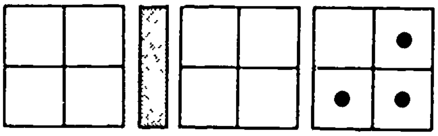
= 7.00

S: $7. or $7.00.



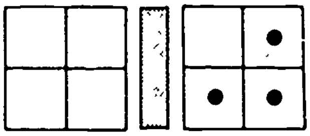
= 0.70

S: Seven dimes are 70¢.



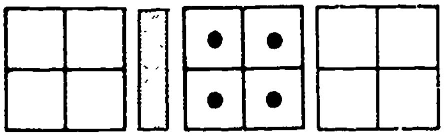
= 0.07

S: Seven pennies are 7¢.

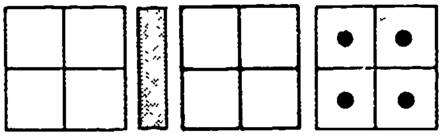


= 0.7 = 0.70

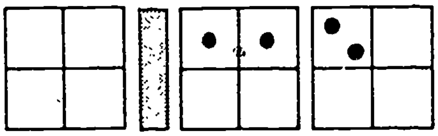
S: Seven dimes are 70¢.

 = 1.50

S: 15 dimes are $1.50.

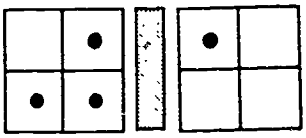
 = 0.15

S: 15 pennies are 15¢.

 = 1.36

S: 12 dimes and 16 pennies equal $1.36.

Create similar problems, as appropriate. Continue with this configuration.

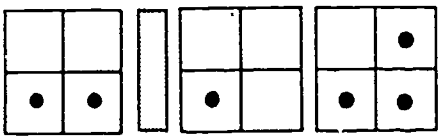
 = 7.8

S: $7.80.

T: What numbers can I put on this Minicomputer by placing exactly one more checker on the dimes' board?

Let students identify and explain how they calculated all four numbers (7.9, 8.0, 8.2, and 8.6).

In a similar manner, ask students for the numbers produced by placing exactly one more checker on the pennies' board of this Minicomputer. (3.28, 3.29, 3.31, and 3.35)

 = 3.27

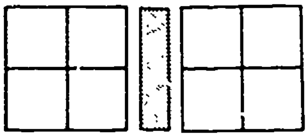
Create similar problems and a worksheet, if you wish.

ACTIVITY D3: COMBINATORIAL PROBLEMS #5

PREREQUISITE: Activity D2

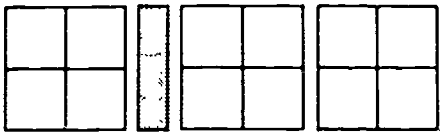
OBJECTIVE: Students will find all numbers that can be shown on the Minicomputer with a specified number of checkers.

Display two Minicomputer boards with a bar between them.



T: What numbers can be put on this Minicomputer by placing exactly two checkers on the dimes' board? (0.2, 0.3, 0.4, 0.5, 0.6, 0.8, 0.9, 1.0, 1.2, 1.6)

Display three Minicomputer boards. This can be either a whole class or an individual activity.



T: What numbers larger than 0.1 can be put on this Minicomputer by placing exactly three checkers on the pennies' board? (0.11, 0.12, 0.13, 0.14, 0.16, 0.17, 0.18, 0.20, 0.24)

Related Activities: W2, W7

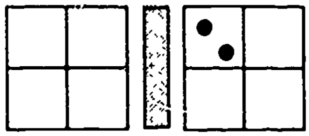
ACTIVITY D4: MISCELLANEOUS PROBLEMS #4

PREREQUISITE: Activity D2

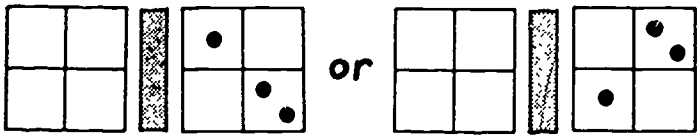
OBJECTIVE: Students will show numbers on the Minicomputer with a specific number of checkers.

Display a ones' board and a dimes' board.

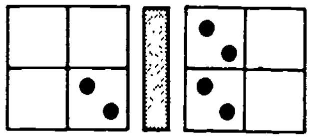
T: Put 1.6 on the Minicomputer with two checkers.



T: Put 1 on the Minicomputer with three checkers.



T: Put 4 on the Minicomputer with six checkers. (Many solutions are possible.)



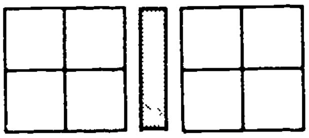
Related Whole Number Activities: W3, W10, W24

ACTIVITY D5: MINICOMPUTER NIM #5

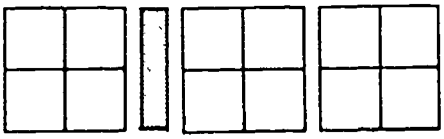
PREREQUISITES: Activities D2 and W14

OBJECTIVE: Students will develop strategies for playing Minicomputer Nim with decimal numbers.

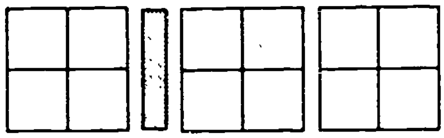
Play Minicomputer Nim with any of the following or similar starting configurations and goals. Refer to Activity W4 for rules to play Minicomputer Nim.



GOAL: 6.7



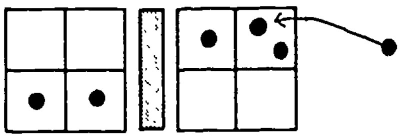
GOAL: 0.79



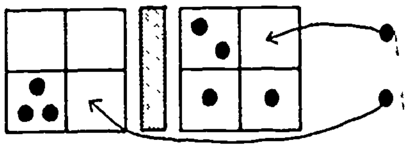
GOAL: 3

After playing several games, present the following and similar problems.

T: What is this number? (4.6)
Put 5 on the Minicomputer with exactly one more checker. (Place a checker on the 0.4-square.)



T. What is this number? (7.9)
Put 9.3 on the Minicomputer with exactly two more checkers. (Place checkers on the 1-square and 0.4-square.)



Related Whole Numbers Activities: W4, W8, W14 and W20

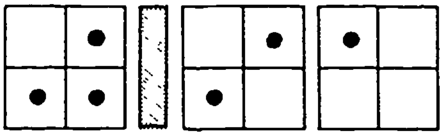
ACTIVITY D6: ADD A CHECKER #5

PREREQUISITES: Activities D2, W15, and D5

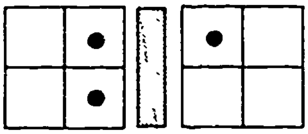
OBJECTIVE: Students will solve problems which involve adding one more checker to a given Minicomputer configuration.

Activity D2 includes examples of problems that involve putting one more checker on the Minicomputer.

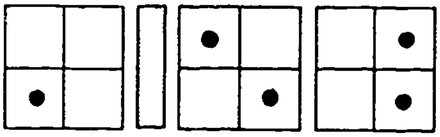
T: What is this number? (7.68)
What numbers can be put on this Minicomputer by placing exactly one more checker on the dimes' board?
(7.78, 7.88, 8.08, 8.48)



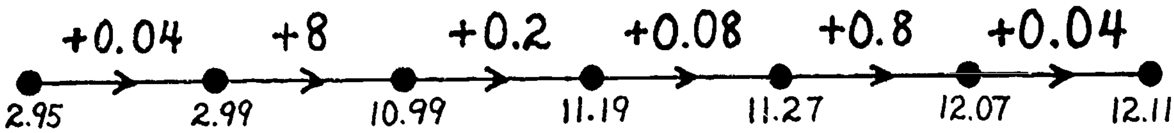
T: What is this number? (5.8)
What numbers can be put on this Minicomputer with exactly one more checker? (5.9, 6.0, 6.2, 6.6, 6.8, 7.8, 9.8, 13.8)



T: What is this number? (2.95)



Following the format in Activity W15, draw this arrow road, one arrow at a time, asking students to place the appropriate checker on the Minicomputer.



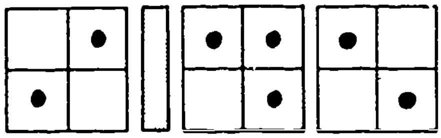
Related Whole Number Activities: W6, W15

ACTIVITY D7: REMOVE A CHECKER #4

PREREQUISITES: Activities D2, W19, D6

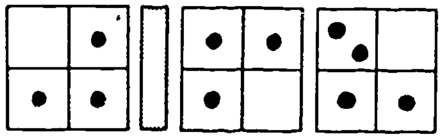
OBJECTIVE: Students will determine the numbers that can be put on the Minicomputer by removing exactly one checker.

T: What is this number? (7.39)

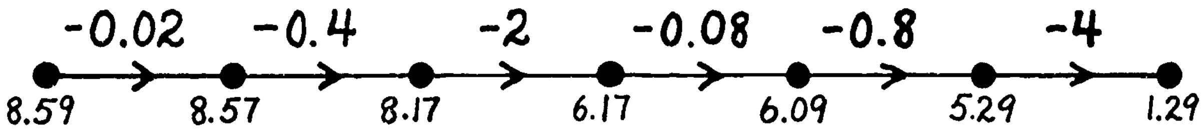


T: What numbers can be put on this Minicomputer by taking off exactly one checker? (3.39, 5.39, 6.59, 6.99, 7.29, 7.31, 7.38)

T: What is this number? (8.59)



Following the format in W15 and W19, draw this arrow road, one arrow at a time, asking students to remove the appropriate checker from the Minicomputer.



Related Whole Number Activities: W9 and W19

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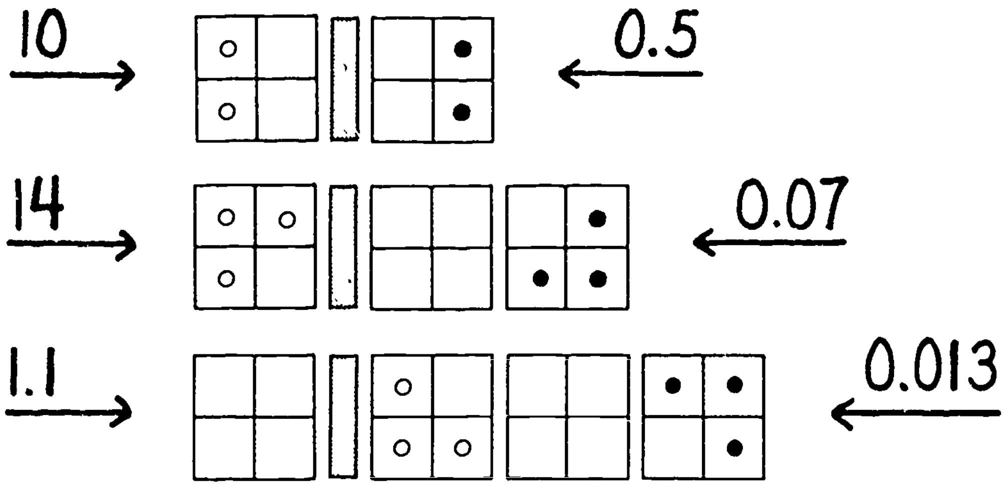
ACTIVITY D8: TUG OF WAR #5

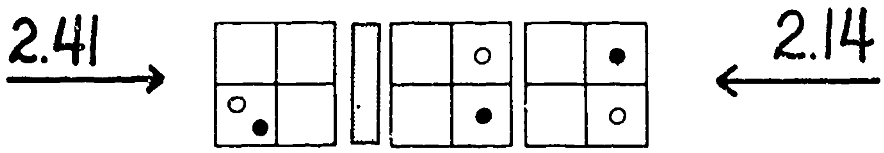
PREREQUISITES: Activities D2 and W17

OBJECTIVE: Students will develop strategies for playing Minicomputer Tug of War with decimals.

Play Minicomputer Tug of War with any of the following or similar starting configurations. Refer to Activity W11 for the rules of Tug of War.



After playing several games, present several problems that might arise when playing Minicomputer Tug of War. For example, put this configuration on the Minicomputer.



T: Blue's number is 2.14 and it is Blue's turn. Find a winning move for Blue. (Move the blue checker on either the 0.1-square or the 0.04 square to the 0.2 square.)

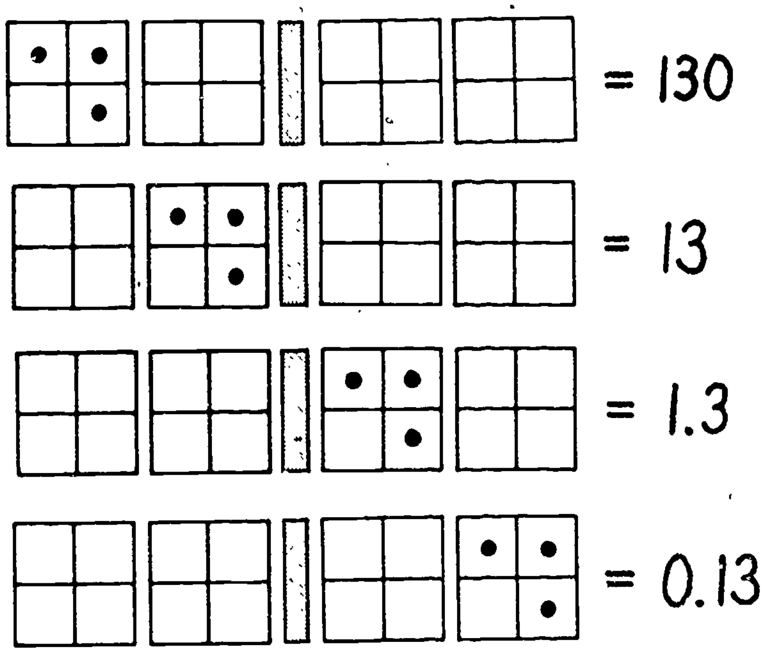
Related Whole Number Activities: W11, W17, W22 and W25

ACTIVITY D9: DECIMAL PATTERNS

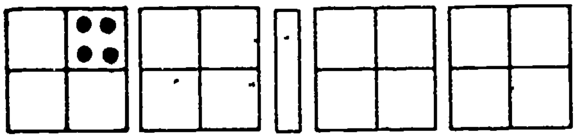
PREREQUISITES: Activities D2 and W21

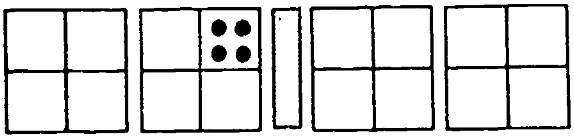
OBJECTIVE: Students will use patterns to solve computational problems involving decimal numbers.

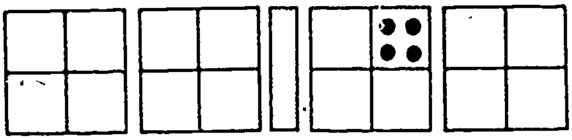
Present this series of configurations, asking students to identify and write each number.

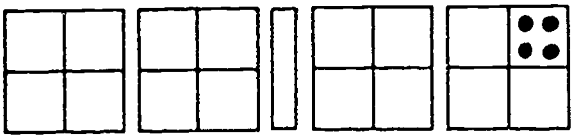


Present this series of configurations, asking students to identify and write each number.

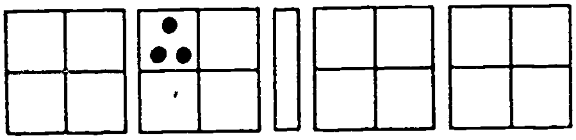
 $= 4 \times 40 = 160$

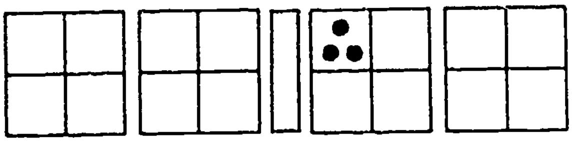
 $= 4 \times 4 = 16$

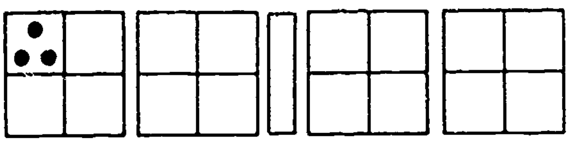
 $= 4 \times 0.4 = 1.6$

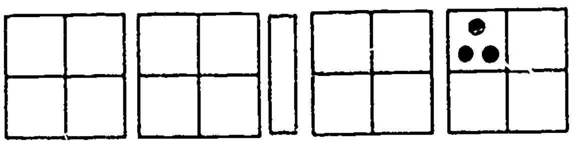
 $= 4 \times 0.04 = 0.16$

Present configurations in an irregular sequence, asking students to identify each number.

 $= 3 \times 8 = 24$

 $= 3 \times 0.8 = 2.4$

 $= 3 \times 80 = 240$

 $= 3 \times 0.08 = 0.24$

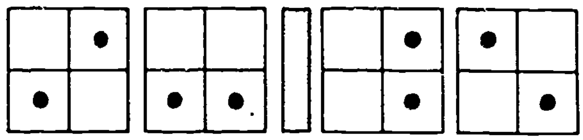
Related Whole Number Activities: W16 and W21

ACTIVITY D10: MOVING A CHECKER #7

PREREQUISITES: Activities W29, D2, D6, and D7

OBJECTIVE: Using mental arithmetic, students will determine the arithmetic effect of moving one checker on the Minicomputer.

T: What is this number? (63.59)

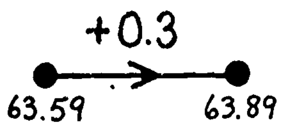


Move a checker from the 0.1-square to the 0.4-square.

T: What number is on the Minicomputer? (63.89) By how much did I increase the number? (0.3) How do you know?

Refer to Activity W23 for a discussion of possible explanations.

Record the answer on an arrow road.



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Continue in a similar manner with the following problems:

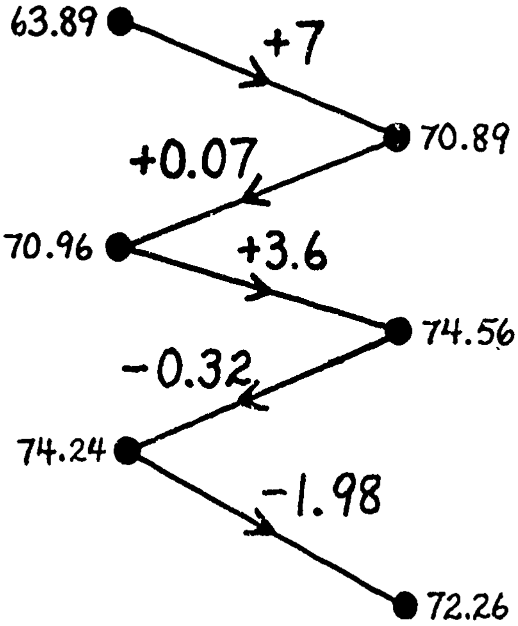
Move a checker from the 1-square to the 8-square.

Move a checker from the 0.01-square to the 0.08-square.

Move a checker from the 0.4-square to the 4-square.

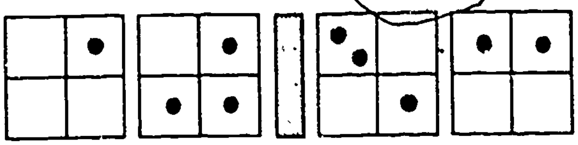
Move a checker from the 0.4-square to the 0.08-square.

Move a checker from the 2-square to the 0.02-square.

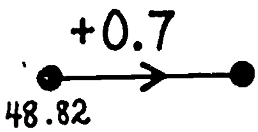


Present problems similar to those above until students can calculate the increase or decrease quite readily. Then change to the following type of problem, where the student must determine the required move on the Minicomputer.

T: What is this number? (48.82)

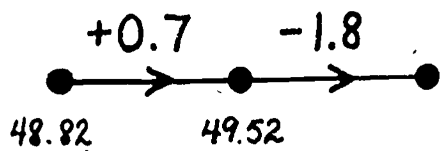


Draw this arrow:



T: 48.82 is on the Minicomputer. Move exactly one checker and make the number 0.7 larger. (Move a checker from the 0.1-square to the 0.8-square.) What number is now on the Minicomputer? (48.82 + 0.7 = 49.52)

Record the answer on the arrow road and present another problem.

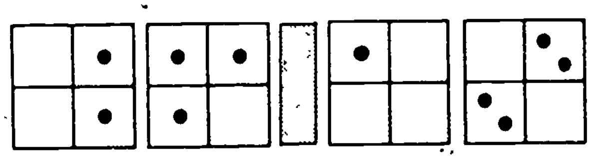


T: Move one checker to make the number 1.8 smaller. (Move a checker from the 2-square to the 0.2-square.) What number is now on the Minicomputer? (47.72)

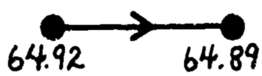
Continue with similar problems. Refer to Activity W29 for similar examples with whole numbers.

Once students can solve the above type of problems readily, proceed to the following problems which are more difficult.

T: What is this number? (64.92)

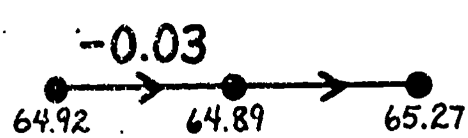


Draw this arrow picture.



T: How much have I increased or decreased 64.92? (0.03 less, since 64.92 - 0.03 = 64.89) Move exactly one checker to put 64.89 on the Minicomputer. (Move a checker from the 0.04-square to the 0.01-square.)

Record the amount of decrease and draw another arrow.



T: How much have I increased or decreased 64.89? (0.38 more) Move one checker to put 65.27. (Move a checker from the 0.02 square to the 0.4-square.)

Continue with similar problems.

Related Whole Number Activities: W23, W26, W29, W31 and W33.

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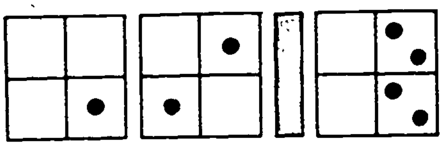
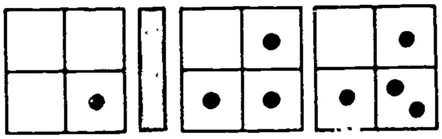
ACTIVITY D11: MINICOMPUTER GOLF #6

PREREQUISITES: Activities W30, D2, D30

OBJECTIVE: Students will develop strategies for playing Minicomputer Golf with decimal numbers.

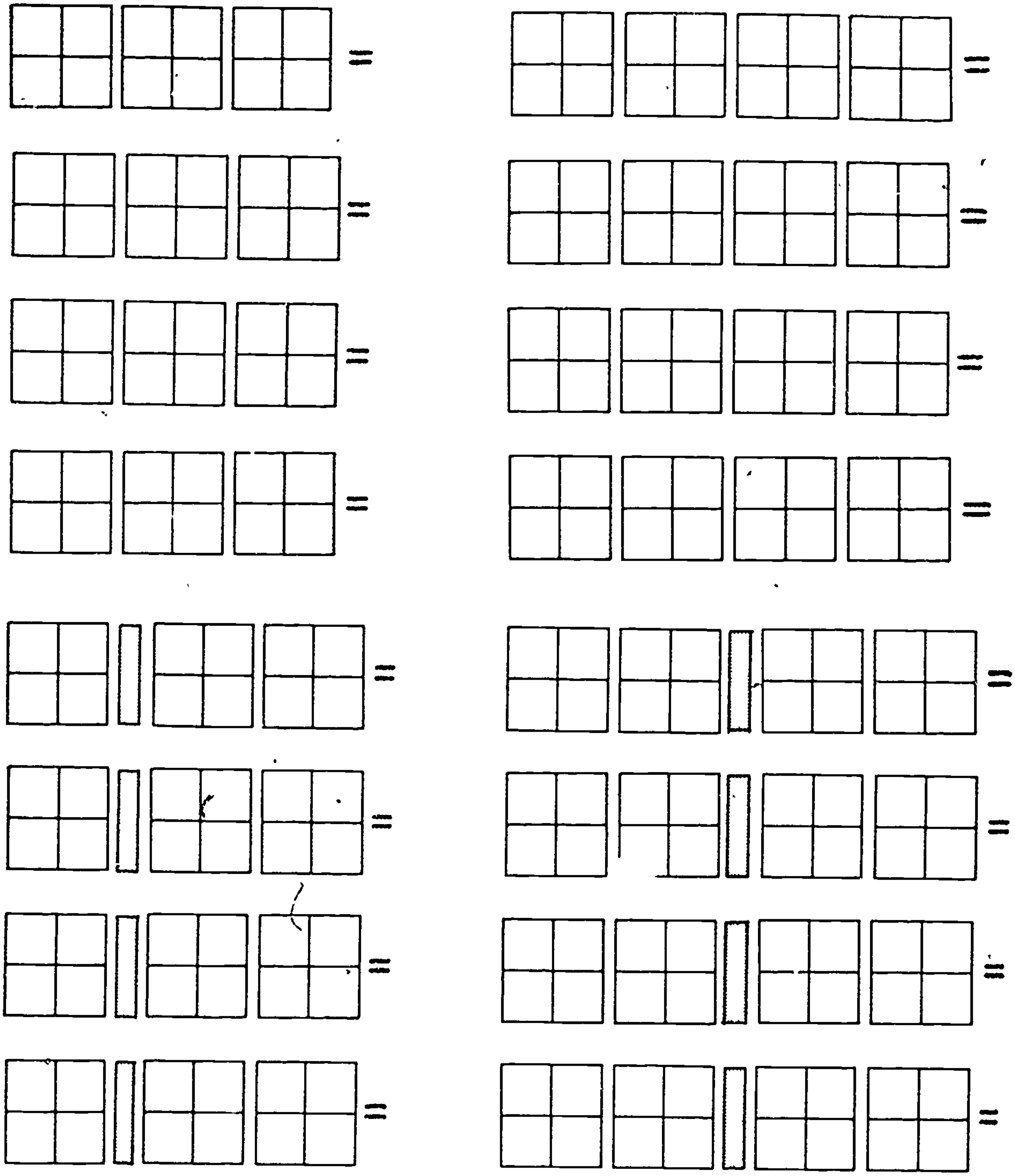
Play Minicomputer Golf with any of the following or similar starting configurations and goals. Refer to Activity W28 for the rules of Golf.

 = 17 GOAL: 31.2

 = 1.78 GOAL: 3

Related Whole Number Activities: W28, W29, W30, W31, W32 and W33.

MINICOMPUTER SHEET



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STRAND III: PROBLEM SOLVING WITH THE HAND-CALCULATOR

INTRODUCTION

Perhaps the most important innovation of modern times for mathematics education is the hand-calculator. Whereas even a few years ago, electronic calculating tools were useful only to specialists, today anyone can have a simple hand-calculator. This ready availability of computing power has profound implications for mathematics curricula at every level. The challenge is to integrate the tool into the curriculum--not as a replacement for learning traditional means of calculations, but as a source of power. Indeed, the power to explore mathematics to such a depth and breadth was never before so readily accessible.

With a tool to process large amounts of routine calculation quickly and accurately, a student can generate a mass of empirical information for experiments with numbers. This opens the world of mathematics to exploration by students just as they might explore scientific phenomena. With the tool facilitating routine calculations, students have the opportunity to develop familiarity with numbers in a variety of activities. Through familiarity comes understanding and growth in a development parallel to the development of computational facility.

One model for hand-calculator activities that promote understanding while practicing computation is given by restricting the use of the keyboard as if the calculator were broken (cf. Activity H9). For example, allowing the use of only $\boxed{5}$, $\boxed{6}$, $\boxed{8}$, $\boxed{9}$, $\boxed{+}$, $\boxed{-}$, $\boxed{\times}$, $\boxed{\div}$ and $\boxed{=}$ to construct numbers provides an opportunity for many activities. Students can be asked to find a way to construct 40. Some might simply press $\boxed{5}\boxed{\times}\boxed{8}\boxed{=}$; others might notice $\boxed{99}$ $\boxed{-}$ $\boxed{59}$ also yields 40. By presenting a sequence of numbers graduated in their difficulty of construction, such as 40, 54, 540, 7, 0.5, we can challenge students' understanding of numbers. Of course, students can be asked to find several solutions to any problem; however, placing a limit on the number of buttons to be pushed gives students an added challenge.

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The activities here assume that each student has a hand-calculator having these two features:

- Algebraic Logic The calculator responds to instructions given in the order in which they are usually written. Pressing $\boxed{2}\boxed{+}\boxed{3}\boxed{\times}\boxed{4}\boxed{=}$ puts 20 on the display.

- Constant Mode Pressing $\boxed{2}\boxed{+}\boxed{3}\boxed{=}\boxed{=}\boxed{=}\boxed{=}$ puts 14 on the display where 3 is the constant.

If your calculators do not operate in a constant mode, let

$\boxed{2}\boxed{+}\boxed{3}\boxed{=}$ ...   mean

$\boxed{2}\boxed{+}\boxed{3}\boxed{=}\boxed{+}\boxed{3}\boxed{=}\boxed{+}\boxed{3}\boxed{=}$ ...

and

$\boxed{5}\boxed{+}\boxed{=}$ ...   mean

$\boxed{+}\boxed{5}\boxed{=}\boxed{+}\boxed{5}\boxed{=}\boxed{+}\boxed{5}\boxed{=}$ ...

Begin any activity with students experimenting with calculators on their own. Encourage a spontaneous and comfortable use of the calculators by your students.

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ACTIVITY H1: CONSTANT FUNCTIONS ON THE HAND-CALCULATOR

PREREQUISITE: Familiarity with negative numbers is optional.

OBJECTIVE: Students will become familiar with the hand-calculator and will practice mental arithmetic.

Provide each student with a hand-calculator. At the beginning of the first lesson, hand out the calculators and allow the students 5 to 10 minutes to explore how they work.

When giving instructions, always do so slowly and clearly. Tell the students to put 13 on the display of their calculators and to press slowly $\boxed{+}\boxed{3}\boxed{=}\boxed{=}\boxed{=}$ and to continue to do so.

T: Which numbers are on the display? (16, 19, 22, 25,...) What do you notice? (The calculator is counting by threes.)

Let the students continue reading the numbers that appear on the display, perhaps until 100 appears.

T: Now clear the display and listen carefully to my instructions. Start with 20 on the display. Press $\boxed{+}\boxed{3}\boxed{=}\boxed{=}\boxed{=}$. What number is on the display? (29)

T: Now press $\boxed{=}\boxed{=}\boxed{=}\boxed{=}$. What is on the display now? (41)

T: Now hide the display. Press $\boxed{=}\boxed{=}\boxed{=}$. What number should be on the display? (50)

Repeat this with $\boxed{=}\boxed{=}\boxed{=}\boxed{=}$. The number 62 will be on the display.

Let several students whisper in your ear or write the number of times they think $\boxed{=}$ should be pressed to go from 62 to 80. Ask one student to respond aloud. (Six times.)

T: What is the smallest number larger than 100 that will appear? (101)

Have a student press $\boxed{=}$ until 101 does indeed appear.

T: Do you think we can make the calculators count backwards by threes?

S: Yes. Press $\boxed{-}\boxed{3}\boxed{=}\boxed{=}\boxed{=}$ and keep doing so.

Have the students try this with the calculator and read the numbers that appear on the display. (98, 95, 92, 89, ...)

Challenge the students to name numbers in this sequence that are smaller than 20, without using their calculators. (17, 14, 11, ...)

If any numbers are disputed, have a student check them with a calculator.

T: What is the smallest positive number that will appear? (2)

T: If we keep pressing $\boxed{=}$ what negative numbers will appear?
(-1, -4, -7, -10,...)

17?

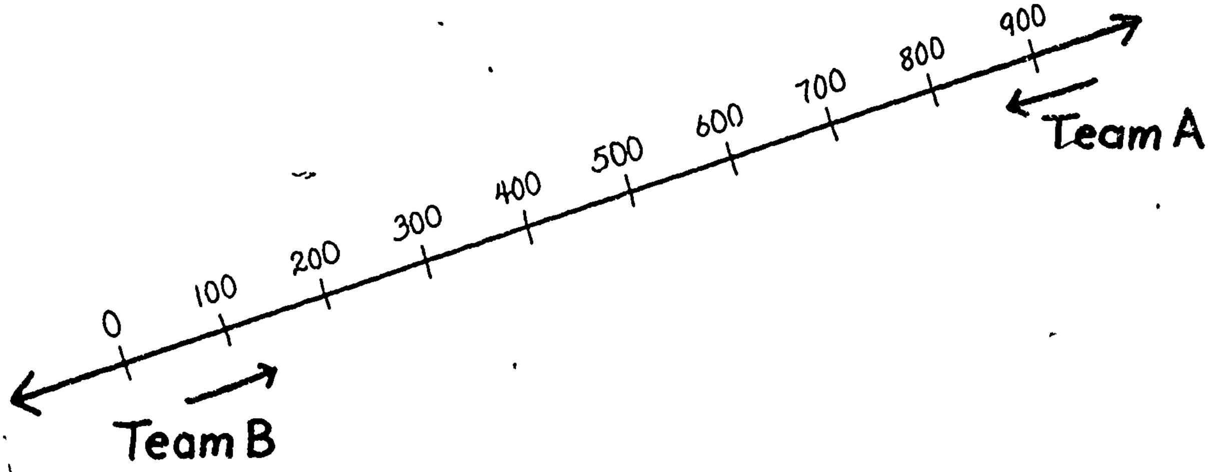
ACTIVITY H2: HAND-CALCULATOR TUG OF WAR

PREREQUISITE: None

OBJECTIVE: Students will practice estimation and mental arithmetic.

Provide each student with a hand-calculator. Allow the students a few minutes to play with and explore the use of the calculator.

Divide the group of students into two teams (A and B) and draw this number line on the board.



T: The students on Team A will start at 167 and the students on Team B at 835.

Let a student from each team mark the approximate location of their team's number on the number line. It might be helpful if the teams use different colors to locate their numbers.

T: The students on Team A can choose only $\boxed{+}$ $\boxed{\text{some number}}$, and the students on Team B only $\boxed{-}$ $\boxed{\text{some number}}$. The teams take turns. The first team to meet the other team's number or to pass it will be the losing team.

T: Each time you make a play, you mark the approximate location of your team's new number on the number line.

Example of a game:

TEAM		DISPLAY	
		167	835
A	$\boxed{+}$ $\boxed{67}$	234	
B	$\boxed{-}$ $\boxed{40}$		795
A	$\boxed{+}$ $\boxed{70}$	304	
B	$\boxed{-}$ $\boxed{200}$		595
A	$\boxed{+}$ $\boxed{20}$	324	
B	$\boxed{-}$ $\boxed{40}$		555
A	$\boxed{+}$ $\boxed{100}$	424	
B	$\boxed{-}$ $\boxed{55}$		500
A	$\boxed{+}$ $\boxed{60}$	484	
B	$\boxed{-}$ $\boxed{10}$		490
A	$\boxed{+}$ $\boxed{5}$	489	

Play this game several times, collectively and/or in one-to-one competition.

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ACTIVITY H3: HAND-CALCULATOR GOLF

PREREQUISITE: None

OBJECTIVE: Students will practice estimation and mental arithmetic.

Provide each student with a hand-calculator. Allow the students a few minutes to play with and explore how the calculator works.

T: We will play Hand-Calculator Golf starting at 237 with a goal of 1,000. Put 237 on the display. The players take turns choosing either $\boxed{+}$ $\boxed{\text{some number}}$ or $\boxed{-}$ $\boxed{\text{some number}}$. We all push the chosen keys and $\boxed{=}$, then we examine the display. The first player to put 1,000 on the display is the winner.

Example:

		Display
First player	$\boxed{+}$ $\boxed{50}$	287
Second player	$\boxed{+}$ $\boxed{287}$	574
Third player	$\boxed{+}$ $\boxed{300}$	874
Fourth player	$\boxed{+}$ $\boxed{300}$	1174
Fifth player	$\boxed{-}$ $\boxed{174}$	1000 (winner)

Play the game many times with starting numbers between 100 and 400 and goals of 1000, 2000, or 3000.

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Variations:

- Allow the students to compete against one another on a one-to-one basis.
- Divide the class into two teams and have team competition.
- Try to reach the goal number in as few steps as possible.
- Try to reach the goal in a specified number of steps. For example, starting at 237 try to reach 1000 in exactly three steps. Here are three solutions:

$\boxed{+}\ \boxed{800}$	$\boxed{+}\ \boxed{700}$	$\boxed{+}\ \boxed{700}$
$\boxed{-}\ \boxed{30}$	$\boxed{+}\ \boxed{70}$	$\boxed{+}\ \boxed{60}$
$\boxed{-}\ \boxed{7}$	$\boxed{-}\ \boxed{7}$	$\boxed{+}\ \boxed{3}$

Such a problem can have an evident relation to place value.

ACTIVITY H4: BUILDING ROADS WITH HAND-CALCULATORS #1

PREREQUISITE: None

OBJECTIVE: Students will practice mental arithmetic and will practice combining arithmetic operations to arrive at a predetermined number.

Ask students to put 7 on their calculator displays. Challenge them to reach 100 with the following restrictions: for each step, they may add or subtract a one-digit number from 1 to 9 or multiply or divide by any of those numbers. For example, $\boxed{-}\boxed{8}$ and $\boxed{\times}\boxed{7}$ are allowed; $\boxed{+}\boxed{6}\boxed{2}$ and $\boxed{-}\boxed{3}\boxed{.}\boxed{5}$ are not allowed.

Draw arrow roads on the board to record student solutions. Two possibilities:

$$7 \xrightarrow{\times 9} 63 \xrightarrow{+9} 72 \xrightarrow{+9} 81 \xrightarrow{+9} 90 \xrightarrow{+9} 99 \xrightarrow{+1} 100$$

$$7 \xrightarrow{\times 8} 56 \xrightarrow{-6} 50 \xrightarrow{\times 2} 100$$

Once you have recorded several student solutions, you can extend the discussion in several directions. How many roads with three arrows from 7 to 100 are there? (Many) Could there be a road with two arrows from 7 to 100? (No)

Additional roads to use for later lessons (negative numbers and decimals are optional, depending on the level of your students):

From 100 to 1

From 17 to 400

From -10 to 50

From -3 to -100

From 1.5 to 80

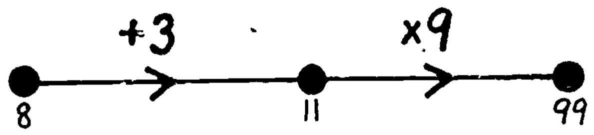
ACTIVITY H5: BUILDING ROADS WITH HAND-CALCULATORS #2

PREREQUISITE: Activity H4

OBJECTIVE: Students will learn and practice efficient mental arithmetic strategies.

In Activity H4 the students built arrow roads from one number to another, using their calculators. This activity continues that exercise but emphasizes finding the shortest possible roads. Before doing this activity the students should have had plenty of practice building roads with little or no emphasis on building the shortest roads. Let them build some roads from 3 to 100 with the 1-digit restriction and ask them to identify the shortest road. (Three arrows) Ask the students to find the shortest possible arrow road from 100 to 3. Eventually the students should decide the answer is also three arrows, and you should have them briefly discuss their reasons.

Now have students use their calculators to find the shortest road from 8 to 99, with the same 1-digit restriction. Record several students' solutions on the board and continue asking for a shorter road. This is one shortest road:



T: How long is the shortest road from 99 to 8? (Two arrows)

Here are some other pairs of numbers for shortest road work:

1, 210	(Three arrows)
-3, 54	(Two arrows)
-40, 50	(Four arrows)

ACTIVITY H6: HAND-CALCULATOR COMBINATORICS

PREREQUISITE: Familiarity with negative numbers and decimals is desirable, but the activity can be adjusted for students who are unfamiliar with these concepts.

OBJECTIVE: Students will practice combining numbers through arithmetic operations. They will be exposed to the concept of equation.

Exercise 1: Counting the Possibilities

Write this information on the board.

$\boxed{7}\ \boxed{}\ \boxed{2}\ \boxed{}\ \boxed{1}\ \boxed{0}\ \boxed{=}$ Za

T: Za is a secret number. One of these symbols (+, −, x, ÷) goes in each blank box. Both boxes can hide the same symbol. Find many possibilities for Za.

Sample solution: $\boxed{7}\ \boxed{x}\ \boxed{2}\ \boxed{\div}\ \boxed{1}\ \boxed{0}\ \boxed{=}$ 1.4

Challenge the students to predict how many numbers Za can be (16) and to find all the possibilities.

Continue with other similar exercises. For example:

$\boxed{1}\ \boxed{2}\ \boxed{}\ \boxed{5}\ \boxed{}\ \boxed{4}\ \boxed{=}$ Mo

$\boxed{8}\ \boxed{}\ \boxed{1}\ \boxed{1}\ \boxed{}\ \boxed{3}\ \boxed{=}$ Si

Exercise 2: Finding the Right Combinations

Write this information on the board.

$\boxed{1}\boxed{2}\boxed{}\boxed{7}\boxed{}\boxed{8}\boxed{=}$ 40

Each blank box may be filled with one of these symbols: +, −, x, ÷. A symbol may be used more than once. Let students use their calculators to solve the problem. ($\boxed{12}\boxed{-}\boxed{7}\boxed{\times}\boxed{8}\boxed{=}$ 40)

Additional problems:

$\boxed{7}\boxed{}\boxed{5}\boxed{}\boxed{3}\boxed{=}$ 4

$\boxed{8}\boxed{}\boxed{3}\boxed{.}\boxed{5}\boxed{}\boxed{4}\boxed{=}$ 18

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ACTIVITY H7: REPEATED-ARROW PICTURES #1

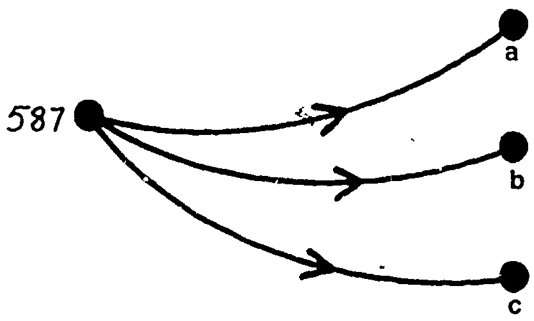
PREREQUISITE: Activity H5

OBJECTIVE: Students will practice mental arithmetic strategies and recog~~nizing~~ numerical patterns.

Exercise 1

Draw this arrow picture on the board.

$\boxed{+}\ \boxed{5}\ \boxed{=}$ ...



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: The arrows represent $\boxed{+}\boxed{5}\boxed{=}\boxed{=}$ ... as many times as you like. (Point to 587 and trace the arrow from 587 to a.) We start with 587 on the display and press $\boxed{+}\boxed{5}\boxed{=}\boxed{=}$ and so on. What numbers could go here (point to a)? (Any integer that ends in 7 or 2 and is more than 587.)

Label a with any correct response.

T: Press $\boxed{=}$ a few more times and watch the numbers that appear on the display. (Point to b.) What numbers could this dot represent?

Let several students give answers. Label b with one of the correct responses.

T (pointing to c): I want to label this dot with a number larger than 1,000. What could it be?

Accept any correct answer and label dot c.

T: What patterns did you notice in the numbers that appear on the display? (The ones' digit is always 2 or 7.)

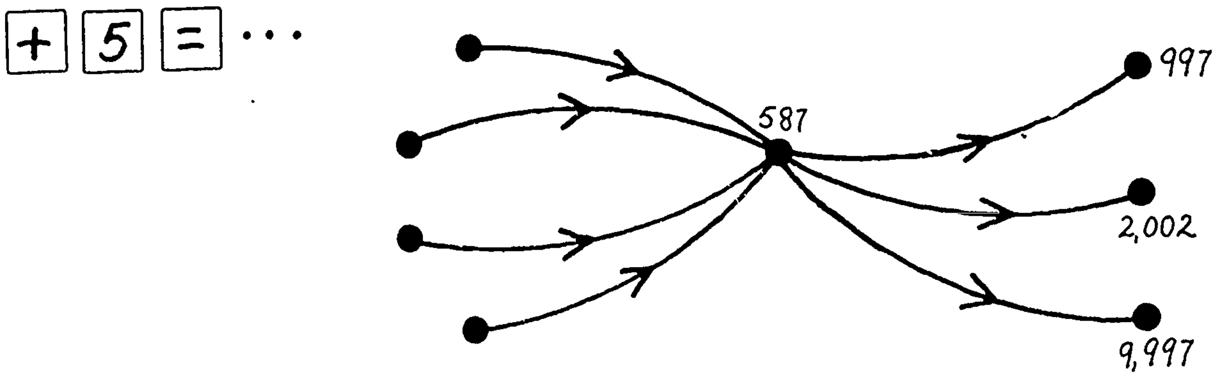
Challenge the students to find a close neighbor of 1,000 that could be in the arrow picture. (997 and 1,002)

Erase the numerals that are at a, b, and c. Label a "997" and ask questions such as the following for b and c.

T: This dot (point to b) is the smallest possible number that is larger than 2,000. (2,002)

This dot (c) is the largest possible number that is smaller than 10,000. (9,997)

Add these arrows to your drawing.



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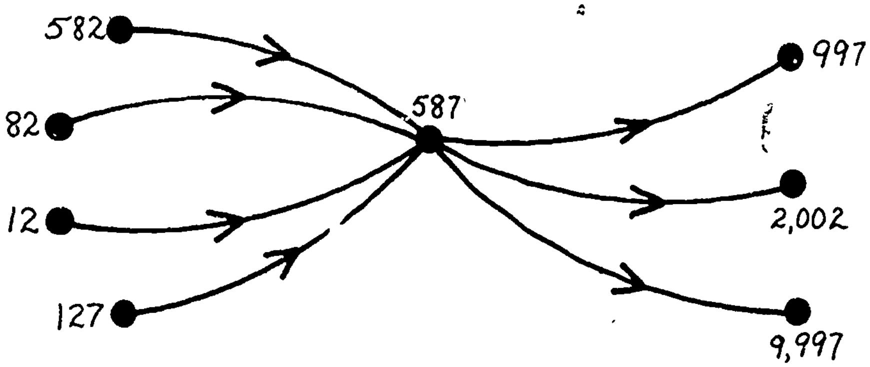
T: What numbers could these dots represent?

Let the students make suggestions. As correct answers are given, record them in the arrow picture. For example:

$\boxed{+}\ \boxed{5}\ \boxed{=}\ \cdots$

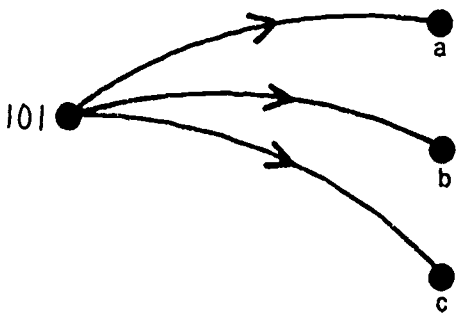


Ask students to name the smallest number larger than 0 that could be in the picture. (2) If the students are familiar with negative numbers, ask them to put some negative numbers in the picture.

Exercise 2

Draw this arrow picture on the board.

$\boxed{+}\ \boxed{4}\ \boxed{=}\ \cdots$



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

Continue as in Exercise 1. A possible dialogue is given here.

T: This dot (point to a) is the closest neighbor to 200. (201)

How many times must $\boxed{=}$ be pressed to get 201 on the display? (25) Could 301 be in this arrow picture? (Yes)

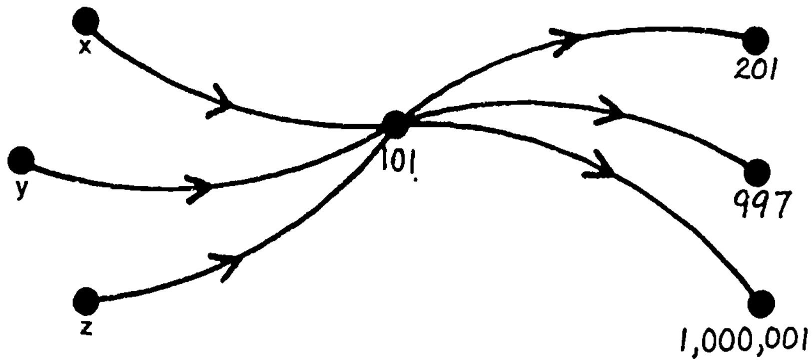
This dot (b) is the largest possible number that is smaller than 1,000. (997)

(Point to c.) What is the closest neighbor to 1,000,000 that could be in the picture? (1,000,001)

Do you notice any patterns in the numbers that appear on the display? (The ones' digit is always odd.)

Extend the arrow picture on the board as follows:

$\boxed{+}\ \boxed{4}\ \boxed{=}\ \cdots$



T (pointing to x): What number could be here?

Let several students answer and label x and y with two of the correct answers.

T: Label this dot (point to z) with the smallest possible number larger than 0. (1)

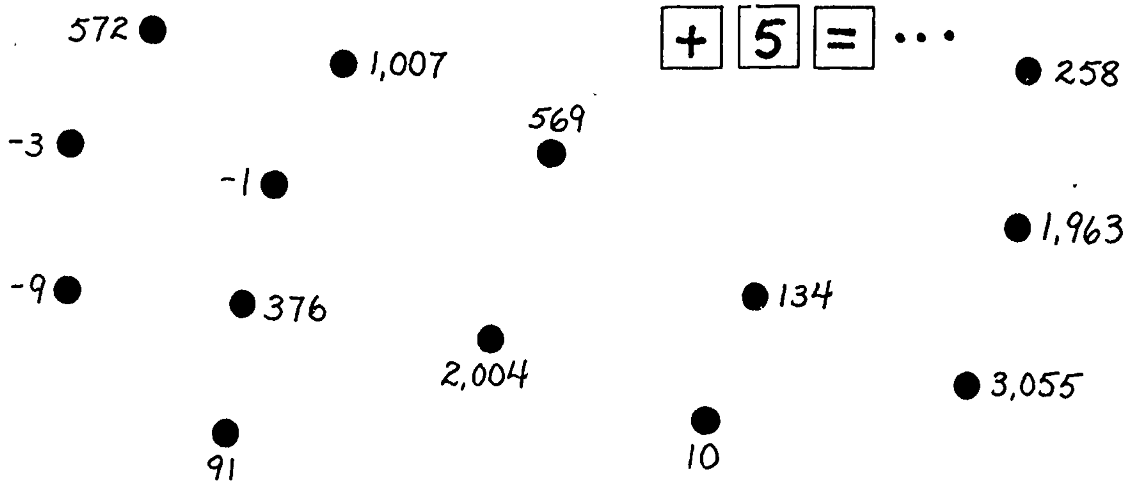
Discuss this answer with the class. Were they able to answer without using the calculators? If so, how?

ACTIVITY H8: REPEATED-ARROW PICTURES #2

PREREQUISITE: Activity H7

OBJECTIVE: The students will use mental arithmetic strategies to identify the numerical patterns that form the basis of modular arithmetic.

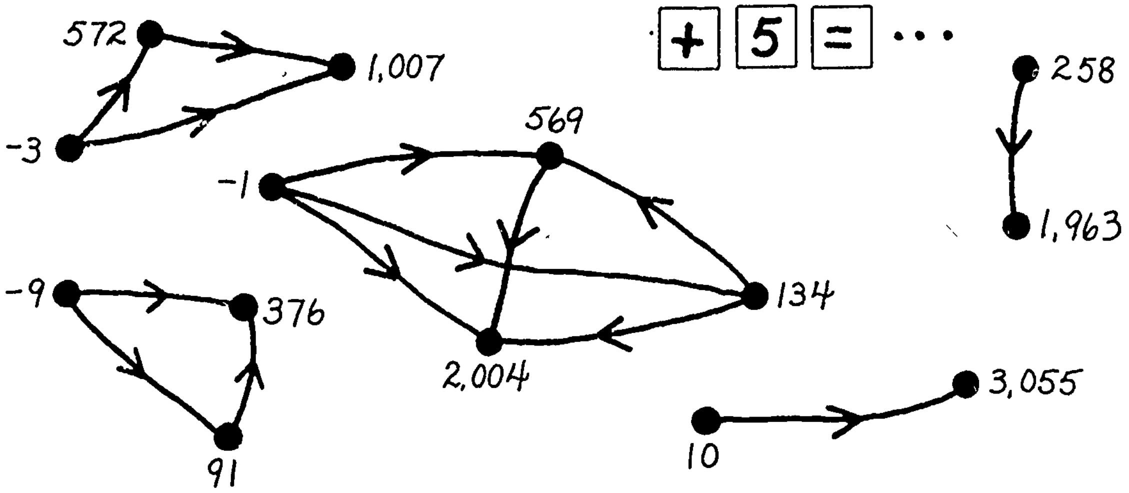
Draw this picture with carefully spaced dots. Negative numbers are optional, depending upon the level and experience of your students.



Note: If possible, prepare this drawing ahead of time on the chalkboard or on an easel-size sheet of paper that can be taped to the board. A careful placement of dots will result in an interesting arrow picture that suggests the many numerical patterns that become apparent when counting by fives.

T: Here is a picture with many dots. The red arrow is for $\boxed{+}\boxed{5}\boxed{=}$...
Put 572 on your calculator display. Press $\boxed{+}\boxed{5}\boxed{=}\boxed{=}$ as many times as you like. Watch the numbers on the display. What do you notice? (All the numbers end in 2 or 7.)

Let the students find a number in the picture on the board that can be connected by a red arrow to 572. Complete the arrow picture using student suggestions. Encourage the use of calculators in the search for missing arrows. The final picture looks like this:



Ask the students what they notice about this picture and discuss the patterns they recognize. For example, if someone points out that there are five separate pieces to the picture, ask them to explain why that might be. Accept any explanation that comes close to acknowledging that there are five distinct + 5 sequences for the integers.

ACTIVITY H9: BUILDING ROADS WITH A BROKEN CALCULATOR

PREREQUISITE: Activity H4

OBJECTIVE: Students will practice mental arithmetic strategies and the combining of arithmetic operations to reach a numerical goal.

Write these symbols on the board.

$\boxed{5}\ \boxed{6}\ \boxed{8}\ \boxed{9}\ \boxed{+}\ \boxed{-}\ \boxed{\times}\ \boxed{\div}\ \boxed{=}$

T: Put 7 on the display. The only keys you may press are $\boxed{5}$, $\boxed{6}$, $\boxed{8}$, $\boxed{9}$, $\boxed{+}$, $\boxed{-}$, $\boxed{\times}$, $\boxed{\div}$, and $\boxed{=}$. So you can add, subtract, multiply or divide by any number whose digits are 5, 6, 8, or 9. Start with 7 and try to put 11 on the display.

Write the following on the board.

START	GOAL
7	11

Give the class a few minutes to explore this situation. Ask for some of their solutions and record them on the board. Sometimes ask the class to check a solution by pressing the suggested sequence of keys on the hand-calculator. It is not necessary to verify every solution.

A few of the many possible solutions are shown here.

START	GOAL
7 + 8 + 5 − 9 =	11
7 − 5 + 9 =	11
7 + 5 9 ÷ 6 =	11

When several solutions have been recorded, continue this activity with other starting numbers and goals. For example:

START	GOAL
8 + 8 + 5 =	21
8 × 6 − 9 − 9 − 9 =	21
8 ÷ 8 + 5 + 5 + 5 + 5 =	21

START	GOAL
43 − 5 − 5 − 5 − 6 × 6 =	132
43 − 9 − 6 − 6 × 6 =	132

Challenge the class to think about efficient solutions by introducing the idea that it costs one cent to press a key and asking for solutions that cost no more than twelve cents. The starting number is free. Later you may wish to permit students to experiment with unfamiliar operations. For example, a solution for building a road from 6 to 3 could be $\boxed{6}\boxed{\div}\boxed{6}\boxed{+}\boxed{8}\boxed{=}\boxed{\sqrt{\ }}$.

For variety, encourage some students to find solutions by using as many different operations as possible (even if more than twelve cents is spent). Several solutions to each problem are provided here. (Problems involving decimals are optional, depending on the experience of your students.)

	GOAL	COST
6 × 8 − 8 − 5 =	35	7¢
6 × 5 + 5 =	35	5¢
6 ÷ 6 + 8 × 5 − 5 − 5 =	35	11¢

	GOAL	COST
5 + 5 ÷ = = × 5 =	0.5	8¢
5 + = = = ÷ 5 ÷ 8 =	0.5	9¢
5 + = ÷ 5 ÷ = =	0.5	7¢

	GOAL	COST
5 ÷ = =	0.2	3¢
5 × 8 ÷ 8 ÷ 5 = =	0.2	8¢
5 ÷ 5 ÷ 5 =	0.2	5¢

ACTIVITY H10: PRODUCING NUMBERS WITH A BROKEN CALCULATOR

PREREQUISITE: Activity H5

OBJECTIVE: The students will increase their understanding of numbers and practice strategies for producing prescribed numbers through combinations of arithmetic operations.

Note: In this activity, negative numbers and decimals are likely to appear on the students' displays. Don't be alarmed. Students who are unfamiliar with these concepts are often enthusiastic learners when a hand-calculator is available.

Exercise 1

Write these symbols on the board.

$\boxed{2}\ \boxed{8}\ \boxed{+}\ \boxed{-}\ \boxed{\times}\ \boxed{\div}\ \boxed{=}\ \boxed{.}$

T: The only keys you may press are: $\boxed{2}$, $\boxed{8}$, $\boxed{+}$, $\boxed{-}$, $\boxed{\times}$, $\boxed{\div}$, $\boxed{=}$, and $\boxed{.}$. If you start with 0 and press exactly five of the above keys, what numbers will appear on the display?

One of the many possible numbers is $\boxed{2}\boxed{8}\boxed{\div}\boxed{2}\boxed{=}$ 14.

When several solutions have been recorded, ask for the largest and smallest possible numbers and so on.

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Exercise 2

Put the following on the board.

$\boxed{5}\ \boxed{6}\ \boxed{8}\ \boxed{9}\ \boxed{+}\ \boxed{-}\ \boxed{\times}\ \boxed{\div}\ \boxed{=}$

T: Start with 0 and use only these keys: $\boxed{5}$, $\boxed{6}$, $\boxed{8}$, $\boxed{9}$, $\boxed{+}$, $\boxed{-}$, $\boxed{x}$, $\boxed{\div}$, $\boxed{=}$. Try to put 40 on the display. Continue with 400, 7, 57, 125, 404, and 8000.

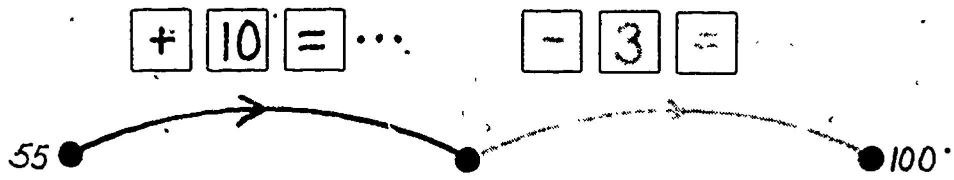
A more difficult list is -7, -60, 0.1, 0.3, and 2.2.

ACTIVITY H11: HAND-CALCULATOR PROBLEMS INVOLVING TWO OPERATIONS #1

PREREQUISITE: Activity H8.

OBJECTIVE: Students will practice skills in pattern recognition and in making and testing hypotheses.

Draw this picture on the board.



T: What number could be here (point to the unlabeled dot)? There are many possibilities. Write at least one on your paper. Try to find a pattern.

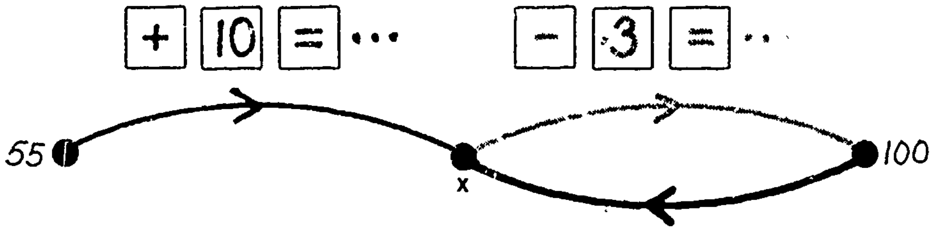
Let the class work on this problem for a few minutes. Then begin to construct a list of students' solutions on the board. Perhaps 115 will be suggested.

T: To get 115, how many times did you press $\boxed{=}$ using the red arrow? (Six) And how many times using the blue arrow? (Five)

As the information is reported, trace the arrows and compute the middle number to check the accuracy of the response. Continue to add numbers to your list as correct suggestions are verified. Note that incorrect suggestions should be checked to verify their incorrectness, just as correct answers are verified. Continue until sufficient numbers are on the list to investigate possible patterns. At this point accept any reasonable explanation, because the idea may be difficult to verbalize. If no explanation is forthcoming, do not be concerned. Do not force an explanation at this early stage. A discussion of the +30 pattern similar to the following is possible.

Suppose that a return arrow for the blue arrow is added to the picture.

Do not write the letter on the board. It is here just to make the description of the lesson easier to follow.



T: What could the green arrow represent? (+3)

The red arrow tells us that x must end in 5 and the green arrow tells us that x must also be 100 plus a multiple of 3. So x may be any number that ends in a 5 and is 100 plus a multiple of 3. A quick check shows that 115 is the smallest such number, that 145 is the next, then 175, and so on.

Once the students recognize the +30 pattern and discuss their explanations for the pattern, change the red arrow to $\boxed{+}\boxed{5}\boxed{=}$... and the blue arrow to $\boxed{-}\boxed{4}\boxed{=}$...

T: Can you guess what the pattern might be now? (+20)

Whether the students make the correct prediction or not, have them use the calculators to find or verify the pattern.

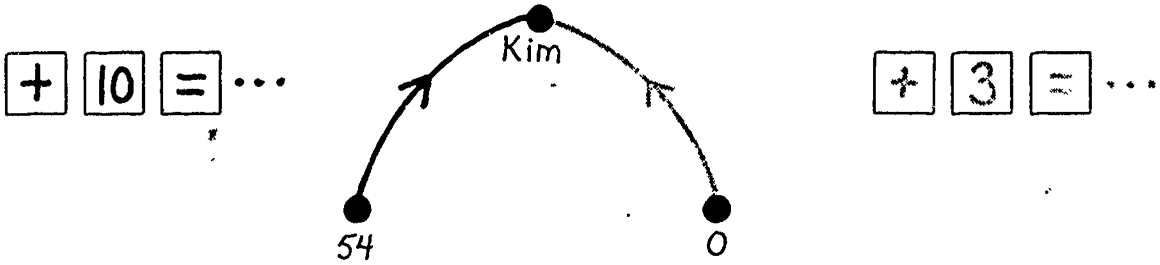
Depending on the level of your class, this lesson can lead to several other lessons that involve hypothesis testing and pattern recognition. For example, what will happen if the 55 and 100 change places? What will the pattern be if, instead of (+10 and -3) or (+5 and -4), the arrows are labeled with a pair of numbers, such as (+8 and -6), which have a common factor?

ACTIVITY H12: HAND CALCULATOR PROBLEM INVOLVING TWO OPERATIONS #2

PREREQUISITE: Activity H11.

OBJECTIVE: Students will practice skills in pattern recognition and in making and testing hypotheses.

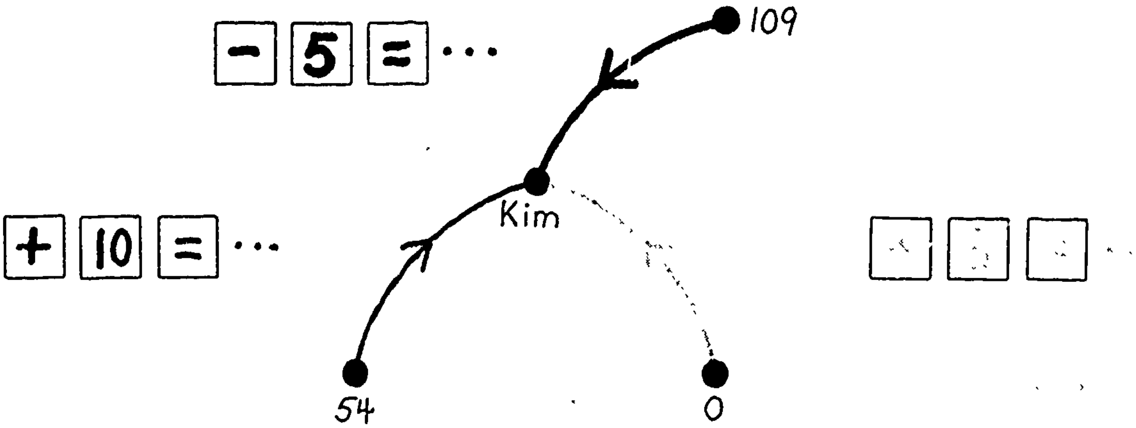
Draw this arrow picture on the board.



T: Kim is in this picture. If you put 54 on your calculator and press $\boxed{+}\boxed{10}\boxed{=}$ ... you will get Kim. Also, if you put 0 on your calculator and press $\boxed{+}\boxed{3}\boxed{=}$... you will get Kim. What are some numbers Kim can be?

Let the students work on their own for a few minutes then begin to record their solutions on the board. Encourage them to find a pattern, so solutions for Kim can be identified without using the calculator. They should conclude that Kim could be 84, 114, 144, 174, 204, 234, and so on.

This can serve as a lesson by itself, but if you want to give a second clue that narrows the candidates for Kim down to one number, extend the arrow picture on the board as follows.



Let the students use the calculators to determine that Kim is 84.

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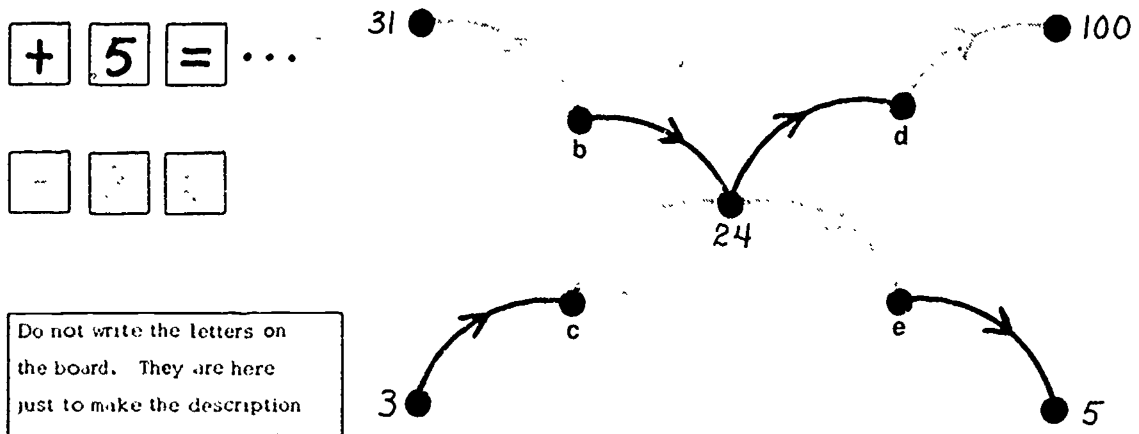
ACTIVITY H13: HAND-CALCULATOR PROBLEMS INVOLVING TWO OPERATIONS #3

PREREQUISITE: Activity H12.

OBJECTIVE: Students will practice skills in pattern recognition and mental arithmetic.

Draw the following arrow picture on the board.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: The red arrow is $\boxed{+}\boxed{5}\boxed{=}$ ... and the blue arrow is $\boxed{-}\boxed{3}\boxed{=}$ .... Use your calculators to determine which numbers the unlabeled dots could represent. Many solutions are possible.

Possible solutions include:

- b: 19, 4, -11, -26, ...
- c: 33, 48, 63, ...
- d: 109, 124, 139, ...
- e: 0, -15, -30, ...

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ACTIVITY H14: HAND-CALCULATOR FUNCTIONS

PREREQUISITE: Activities H10 and H13.

OBJECTIVE: Students will become familar with the concept of function.

Write these symbols on the board.

$\boxed{5}\ \boxed{6}\ \boxed{8}\ \boxed{9}\ \boxed{+}\ \boxed{-}\ \boxed{\times}\ \boxed{\div}\ \boxed{=}$

T: The only keys you may press are $\boxed{5}$, $\boxed{6}$, $\boxed{8}$, $\boxed{9}$, $\boxed{+}$, $\boxed{-}$, $\boxed{x}$, $\boxed{\div}$, $\boxed{=}$. Let's find a method for doubling any starting number you put on the display. Remember that the method must work for any starting number. Put 27 on the display and find a way to double it using only these keys.
($\boxed{27}\ \boxed{+}\ \boxed{=}$ 54)

Once the students have found the solution, ask them to find a way to multiply by 3 ($\boxed{+}\ \boxed{=}\ \boxed{=}$) and by 4 ($\boxed{+}\ \boxed{=}\ \boxed{=}\ \boxed{=}$ or $\boxed{+}\ \boxed{=}\ \boxed{+}\ \boxed{=}$).

T: Now find a way to put one half of any starting number on the display.
($\boxed{+}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{\div}\ \boxed{8}\ \boxed{=}$ or $\boxed{+}\ \boxed{=}\ \boxed{=}\ \boxed{\div}\ \boxed{6}\ \boxed{=}$)

Find a way to multiply any number by 10 using only these keys.
($\boxed{+}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}$ or $\boxed{+}\ \boxed{=}\ \boxed{x}\ \boxed{5}\ \boxed{=}$)

Problem Solving Through Detective Stories

TABLE OF CONTENTS

STRAND IV: PROBLEM SOLVING THROUGH DETECTIVE STORIES

INTRODUCTION

A detective story comprises a collection of clues to the identity of a secret number. These stories are ideal for reviewing many concepts because they can employ many devices, and in each instance they present them in a varied context. Thus each experience is new and unique and reviews and reinforces arithmetic processes. Detective stories were introduced in the development of the elementary mathematics curriculum of the Comprehensive School Mathematics Program in which they elicited a positive response from students.

Providing students clues to solve a detective story introduces an element of suspense, which can be a powerful motivation in learning to use and to become fluent in the mathematics. By inducing students to follow clues, a detective story encourages habits of concentration and perseverance. The large variety of approaches and clues that can be used allows tailoring to the mathematical needs of any specific group.

Carefule consideration of each clue in a detective story is important. Record information on the board as it is deduced. When the first clue is presented, it is usually appropriate to list all of the possibilities on the board or to indicate a sequence of possibilities--for example, 11, 22, 33, 44, ... Information from subsequent clues can be related to the numbers in this list. Erase or cross out numbers as they are eliminated.

When presenting a detective story to a class, offer a clue for students to discuss. This is an opportunity for them to experience problem solving as a group activity. Only offer a second clue when there is no more fruitful discussion forthcoming. Note that all suggestions are worth examining. Rejection or acceptance of a number must be determined by a mathematical appeal to the clue. In either case, the class is involved in the mathematics of the clue. Your role is to enable the class to work together to find the secret number.

ERIC

The twenty-five detective stories in this strand employ four types of clues: string pictures, arrow pictures, Minicomputer problems, and handcalculator problems.

String Pictures

The information provided by a string picture should be discussed and numbers that fit the clue should be either circled or listed. In the discussion, it is useful to locate in the string picture all numbers suggested. Then, even if a number cannot be the secret number, the class has the opportunity to practice their skills by testing the suggestion against the string labels.

Throughout these activities, we will refer to the left string as "red" and the right string as "blue". If there is a third string, it will be referred to as "green".

Arrow Pictures

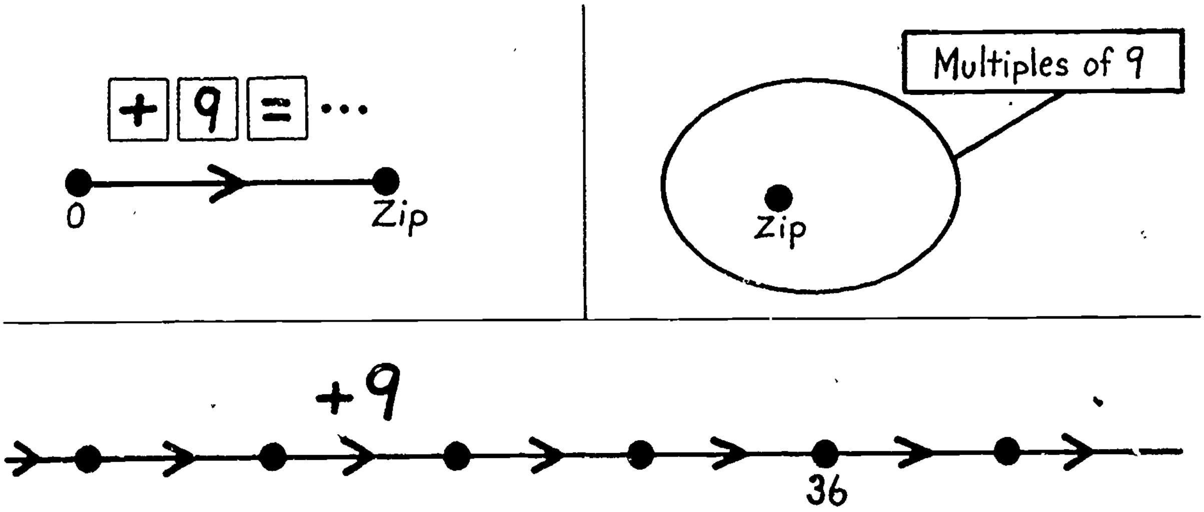
Draw the arrow picture on the board and ask the students to label all of the dots on a copy that they draw themselves or that you have prepared for them. In the illustrations, numbers inside boxes are provided for your information, not to be included in your drawing. The colors of the arrows in the illustrations are indicated by a thin black line for red, a gray line for blue, and a thick black line for green. As students finish labeling their drawings, ask volunteers to label the dots on the board.

Minicomputer

Display the appropriate equipment and invite students to show solutions at the board as part of the discussion.

Hand-Calculator

We encourage using hand-calculators when suggested. However, if they are not available, some hand-calculator clues can be changed to string or arrow picture clues. For example, these three pictures are interchangable as clues since they all tell us that Zip is a multiple of 9.



You may to construct detective stories to meet the specific needs of your class. In general, three to five clues will yield a story of good length. The clues can be adjusted to focus on any operations, relations, or attributes of numbers. Varying the type of clue lends interest to a story, although you may focus on one type of clue to direct student attention to the techniques of that type. Writing a good detective story is an interesting exercise. One of the challenges lies in pitching the difficulty of the clues slightly above the ability of the students. Alternating more and less difficult clues gives a break in the tension of examining the clues and at the same time helps to meet the needs of your group. The stories of this strand all include clues and situations of varied difficulty. Part of your responsibility in guiding the class solution of a detective story is to permit everyone to participate, to allow each to contribute to the solution at a comfortable level of sophistication.

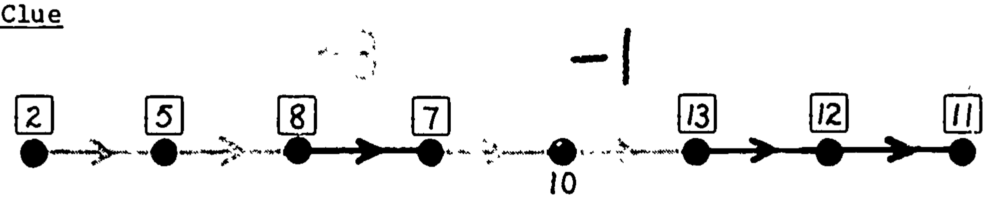
ACTIVITY A1: WHO IS KIWI?

PREREQUISITE: S2, W2

First Clue

T: Kiwi is a secret number that can be put on the ones' board of the Minicomputer with exactly two regular (positive) checkers. Which numbers could Kiwi be? (2, 3, 4, 5, 6, 8, 9, 10, 12, or 16)

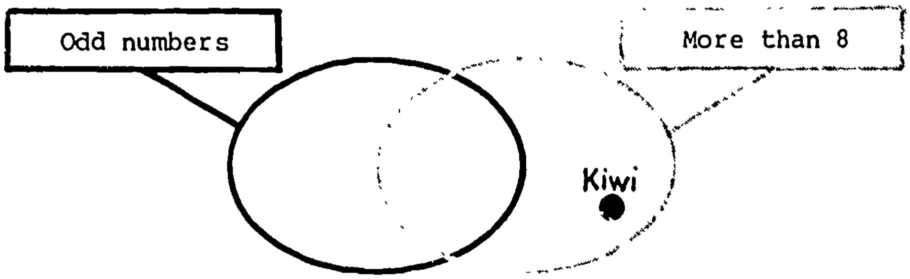
Second Clue



T: Kiwi is in this arrow picture. All the blue arrows are for +3 and all the red arrows are for -1. This number (point to the dot for 7) plus 3 (trace the arrow) is 10. What is this number? (7) We can label this dot 7 because 7 + 3 = 10.

Continue until all the dots are labeled. K.wi could be 2, 5, 8, 10, or 12.

Third Clue



T: What does the red string tell us about Kiwi? (Kiwi is not an odd number, so Kiwi is an even number.) Which numbers could Kiwi be? (2, 8, 10, 12) What does the blue string tell us about Kiwi? (Kiwi is more than 8.) Which numbers could Kiwi be? (10 or 12) Could Kiwi be 8? (No, because 8 is not more than 8.)

ERIC

Fourth Clue

T: Kiwi can be put on the Minicomputer with three checkers on the same square. Which number is Kiwi? (12)

ERIC

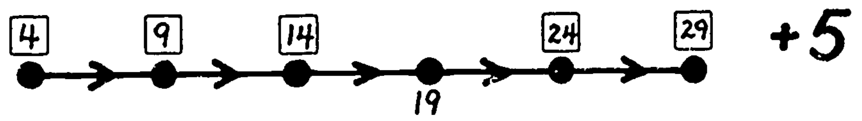
ACTIVITY A2: WHO IS KAWA?

PREREQUISITE: S2, W7

First Clue

T: Kawa can be put on the ones' board of the Minicomputer with exactly three regular (positive) checkers. Which numbers could Kawa be? (3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 20, 24)

Second Clue



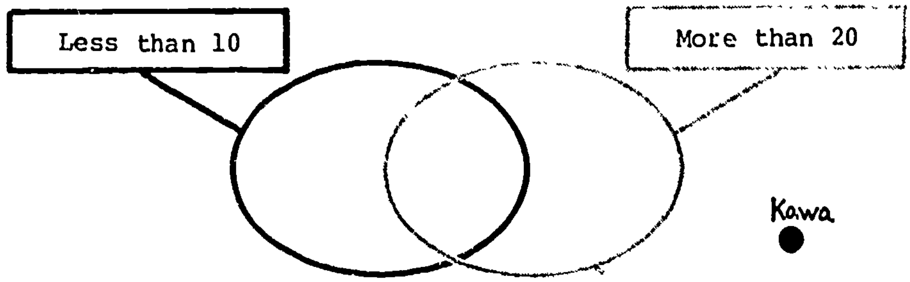
T: Kawa is one of the numbers in this arrow picture. The red arrows are for +5. (Trace the arrow that starts at 19.) Which number is 19 + 5? (24)

Label the dot for 24. Trace the arrow that ends at 19.

T: Which number +5 is 19? (14) Since 14 + 5 = 19, we will label this dot 14.

Continue until all the dots are labeled. Kawa could be 4, 9, 14, or 24.

Third Clue



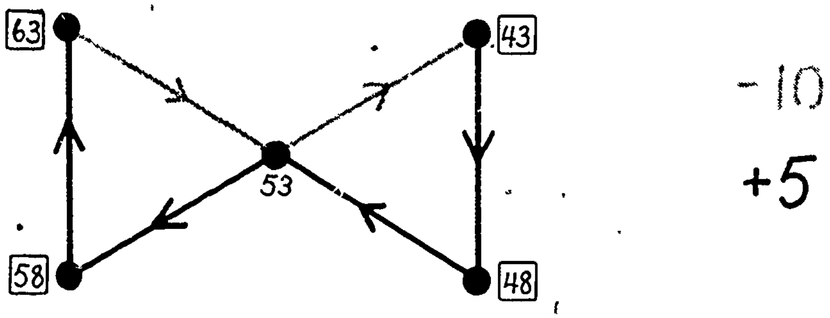
T: What does the red string tell us about Kawa? (Kawa is not less than 10.) Which numbers could Kawa be? (14 or 24) What does the blue string tell us about Kawa? (Kawa is not more than 20 so Kawa can not be 24.) Which number is Kawa? (14)

ERIC

ACTIVITY A3: WHO IS FLUFF?

PREREQUISITE: W5

First Clue



T: Fluff is one of the numbers in this arrow picture. The red arrows are all for +5 and the blue arrows are both for -10. Label the dots to find out which numbers can be Fluff.

Second Clue

T: Fluff can be put on the Minicomputer with exactly two checkers. Which number is Fluff? (48)

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ACTIVITY A4: WHO ARE TIC AND TAC?

PREREQUISITE: A2

First Clue

T: Tic and Tac are whole numbers. We can see them in this arrow picture. What are the red arrows for? (+5) Which number is larger? (Tac) How much larger is Tac than Tic? (100) How do you know?

Perhaps your students will not know how much larger Tac is at this time. If necessary, return to this question after completing the table below.

T: If Tic were 10, what would Tac be? (110)
If Tic were 100? (200)

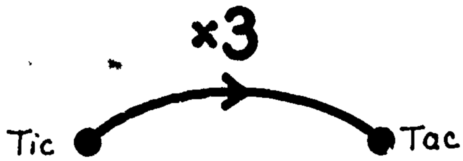
Continue this activity and record the information in a table like the one below.

Tic	Tac
10	$\boxed{110}$
100	$\boxed{200}$
20	$\boxed{120}$
50	$\boxed{150}$
27	$\boxed{127}$
184	$\boxed{284}$
$\boxed{261}$	361

Draw a blue arrow from Tic to Tac and ask what the blue arrow could represent. (+100) Write "+100" in blue near the blue arrow and above the table.

Note: The blue arrow could represent other relationships but in this activity we are only interested in +100.

<u>Second Clue</u>



T: What does this green arrow tell us about Tic and Tac? (Tic x 3 = Tac.)
Let's make another table with numbers that fit this clue for Tic and Tac.
If Tic were 5, what would Tac be? (15)

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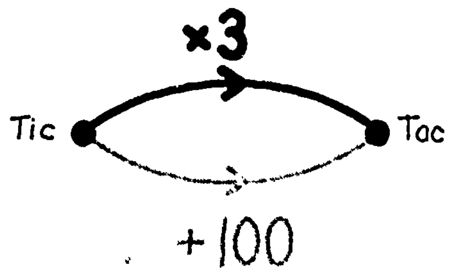
Continue this activity and record the resulting information in a table similar to the one below.

Tic	Tac
5	$\boxed{15}$
100	$\boxed{300}$
31	$\boxed{93}$
120	$\boxed{360}$
251	$\boxed{753}$
42	126

Extend the second arrow picture as follows.



T: Tic x 3 = Tac, and Tic + 100 = Tac. What numbers are Tic and Tac? (Tic is 50 and Tac is 150.)

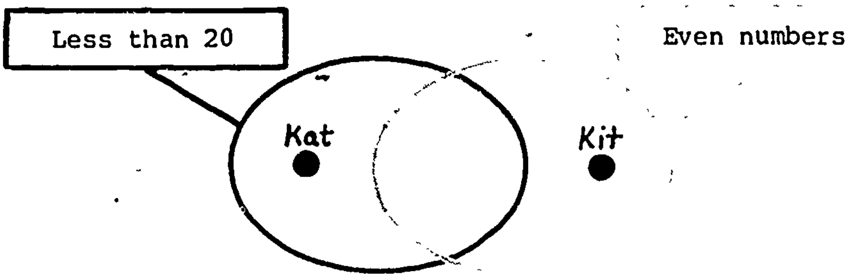
If necessary, direct the students to consider the entries in the two tables to determine if any of them would fit in both tables.

ERIC

ACTIVITY A5: WHO ARE KAT AND KIT?

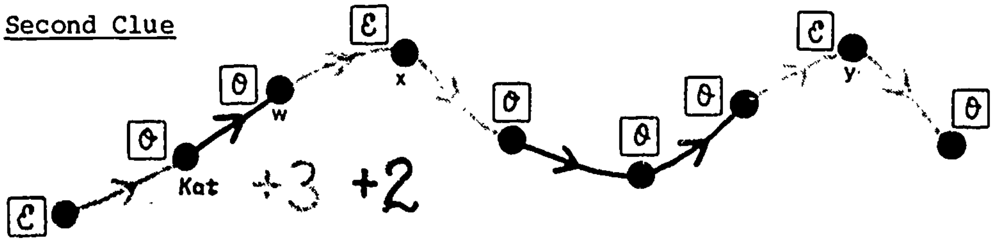
PREREQUISITE: S2, A4

First Clue



T: Kat and Kit are whole numbers. What does this string picture tell us about Kat? (Kat is an odd number less than 20.) What are some numbers Kat could be? (19, 17, 15, 13, ...) What does this string picture tell us about Kit? (Kit is an even number that is not less than 20.) What are some numbers Kit could be? (20, 22, 24, 26, ...)

Second Clue



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: Where is Kit in this arrow picture? (Kit is one of the dots to the right of Kat.) We know that Kit is more than Kat. What else do we know about Kit? (Kit is an even number.) Let's see if we can determine which of the numbers in this arrow picture are even. Is Kat even or odd? (Odd)

Write " θ " near the dot for Kat. Point to w.

T: Does this dot represent an even number or an odd number? (An odd number, because an odd number +2 is always an odd number.)

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If necessary, select several odd numbers and determine that the number that is 2 more than each of them is also an odd number. Write " 𝒪 " near w.

T (pointing to x): Is this number even or odd? (Even, because an odd number +3 is always even.)

If necessary, test several odd numbers and determine that the numbers that are 3 more than each of them are all even numbers. Write " ℰ " near x. Continue this activity until all the dots are labeled either " ℰ " or " 𝒪 ". Conclude that there are two dots that could be Kit.

Third Clue

T: Kit is the largest even number in this arrow picture. Where is Kit?

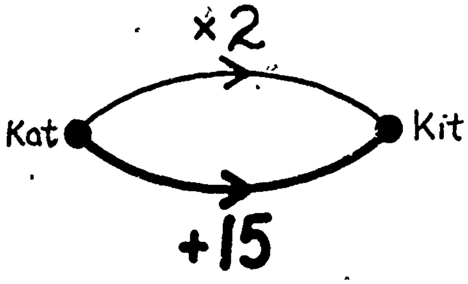
Label dot y "Kit". Draw a green arrow from Kat to Kit.

T: What could the green arrow represent? (+15) How much larger is Kit than Kat? (15) Which numbers could Kat and Kit be?

Write "+15" in green near the green arrow. Encourage the students to offer many suggestions for Kat and Kit. As necessary, remind them of the first clue. Record their suggestions in a table. All the possible pairs are recorded in the table below for your information.

Kat	Kit
5	20
7	22
9	24
11	26
13	28
15	30
17	32
19	34

Fourth Clue



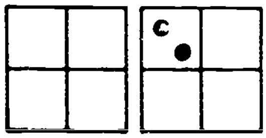
T: What does this arrow picture tell us? (Kat x 2 = Kit, and Kat + 15 = Kit.) Which numbers are Kat and Kit?

Encourage the students to write down their answers so you can check them. Consider at least one incorrect solution. For example, assume Kat is 9 and show that 9 x 2 $\neq$ 9 + 15. Conclude that Kat is 15 and Kit is 30.

ACTIVITY A6: WHO IS ZIP?

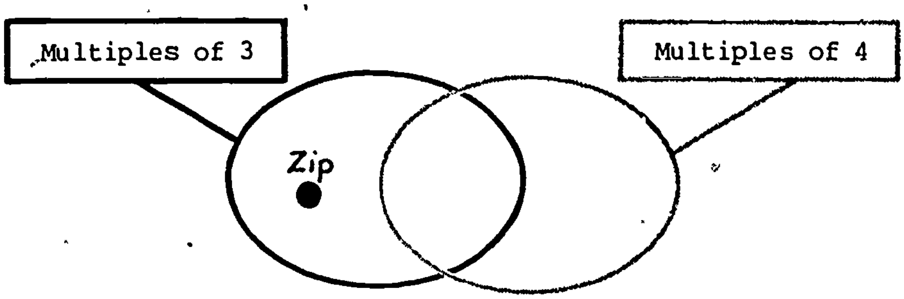
PREREQUISITE: S8, W6

First Clue



T: Zip can be put on this Minicomputer with one more regular (positive) checker. Which numbers could Zip be? (17, 18, 20, 24, 26, 36, 56, 96)

Second Clue



T: What does the red string tell us about Zip? (Zip is a multiple of 3.) Which of the numbers in our list are multiples of 3? (18, 24, 36, 96) What does the blue string tell us about Zip? (Zip is not a multiple of 4.) Which of the numbers in our list could be Zip? (18)

ERIC

ACTIVITY A7: WHO IS SNIFF?

PREREQUISITE: S2, W11

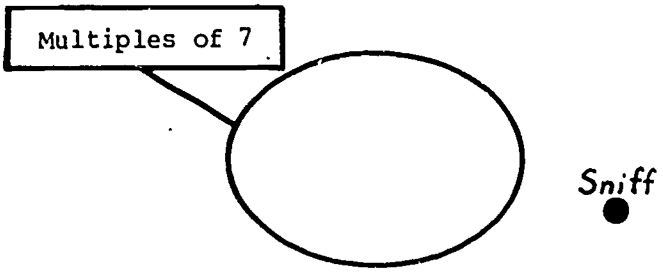
First Clue

T: Sniff can be put on the Minicomputer with exactly one regular (positive) checker on the tens' board and one regular (positive) checker on the ones' board. Which numbers could Sniff be?

As possibilities for Sniff are suggested, list them on the board in a systematic way. This will help the students find all the possible numbers. For example:

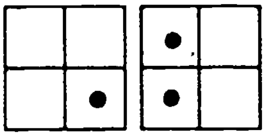
11	21	41	81
12	22	42	82
14	24	44	84
18	28	48	88

Second Clue



T: What does this string picture tell us about Sniff? (Sniff is not a multiple of 7.) Which numbers can we cross out? (14, 21, 28, 42, 84)

Third Clue

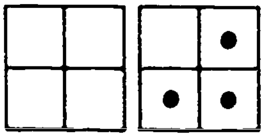


T: Sniff can be put on this Minicomputer by moving exactly one checker that is now on the ones' board. Which numbers could be Sniff? (22)

ACTIVITY A8: WHO IS TIP?

PREREQUISITE: S2, W11, A2

First Clue

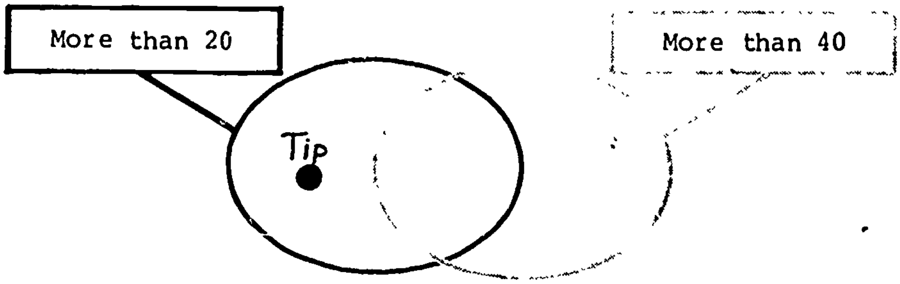


T: Tip can be put on this Minicomputer by moving one of these checkers to the tens' board. Which numbers could Tip be?

As possibilities for Tip are suggested, list them on the board in a systematic way. This will help the students to find all of the possible numbers. For example:

16	26	46	86
15	25	45	85
13	23	43	83

Second Clue



T: What does this string picture tell us about Tip? (Tip is more than 20 and less than or equal to 40.) Which numbers could be Tip? (23, 25, 26)

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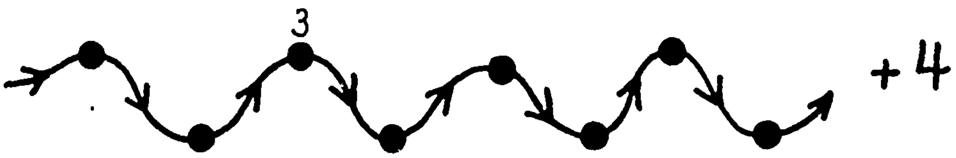
Third Clue



Note: If your students are familiar with negative numbers, change "3" to "-5" in the arrow road.

T: What are the red arrows for? (+4) This arrow road has no beginning and no end. It goes on forever in both directions. Tip is one of the numbers in this arrow road. Which numbers could Tip be? (23)

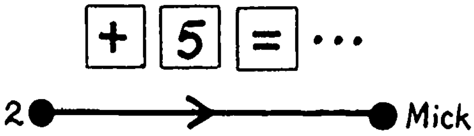
Your students may suggest one of these methods for determining Tip:

- Extending the arrow picture;
- Counting by 4's;
- Using a hand-calculator, $\boxed{3}\,\boxed{+}\,\boxed{4}\,\boxed{=}$ ... or $\boxed{-5}\,\boxed{+}\,\boxed{4}\,\boxed{=}$ ...; or
- Observing that Tip is one less than a multiple of 4, and therefore Tip is 24 - 1 or 23.

ACTIVITY A9: WHO IS MICK?

PREREQUISITE: S8, H1

First Clue



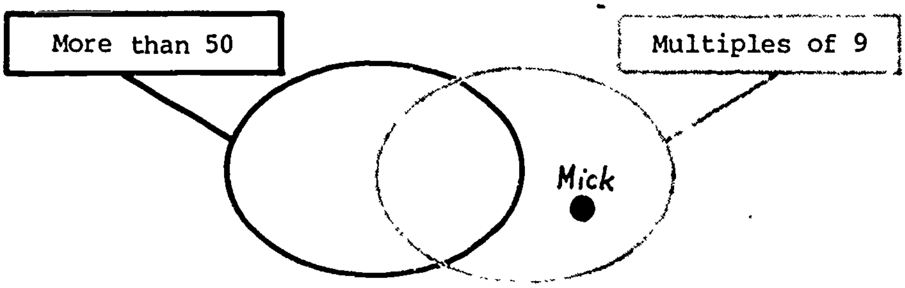
T: This red arrow tells us to put 2 on the display of the calculator and press $\boxed{+}\boxed{5}\boxed{=}$ .... One of the numbers that will appear on the display is Mick.

Allow the students to experiment for a few minutes.

T: What patterns have you observed in the numbers that appear on the display? (The ones' digit of every number is either 2 or 7.) Which numbers could Mick be? (7, 12, 17, 22, 27, 32 ...)

List the possible numbers up to 52 on the board. Indicate with "..." that the list is infinite.

Second Clue

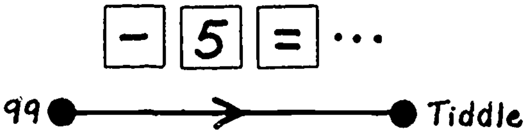


T: What does the red string tell us about Mick? (Mick is 50 or less, so we can cross out all the numbers more than 50 in our list.) What does the blue string tell us about Mick? (Mick is a multiple of 9.) Which number is Mick? (27)

ACTIVITY A10: WHO IS TIDDLE?

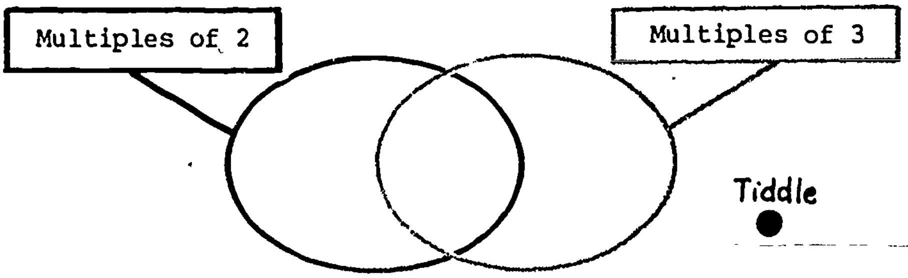
PREREQUISITE: S8, W9, A9

First Clue



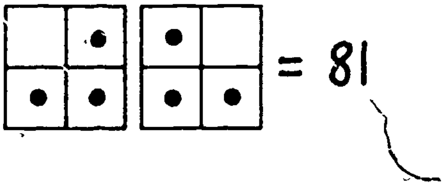
T: Which numbers could Tiddle be? (94, 89, 84, 79, 74, 69, ...)

Second Clue



T: What does the red string tell us about Tiddle? (Tiddle is an odd number, so Tiddle could be 89, 79, 69, 59, 49, ...)
What does the blue string tell us about Tiddle? (Tiddle is not a multiple of 3.) Which numbers could Tiddle be? (89, 79, 59, 49, ...)

Third Clue



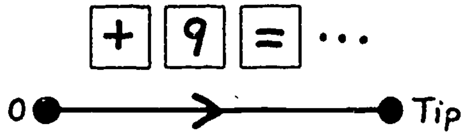
T: Tiddle can be put on this Minicomputer by removing exactly two checkers. Which number is Tiddle? (59)

225

ACTIVITY A11: WHO IS MAX?

PREREQUISITE: S8, W13, A9

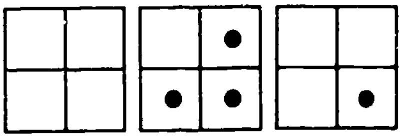
First Clue



T: What does this arrow picture tell us about Max? (Max is a multiple of 9.) What are some of the numbers Max could be? (9, 18, 27, 36, ...) Do you notice any patterns? (The ones' digit and tens' digit of each number have a sum of 9.) Is this always true?

Encourage the students to continue pressing = until 99 appears on the display and to observe that 9 + 9 = 18. Continue pressing = until 189 appears and observe that 1 + 8 + 9 = 18. Perhaps a student will suggest that the sum of the digits of a multiple of 9 is a multiple of 9. Encourage this suggestion.

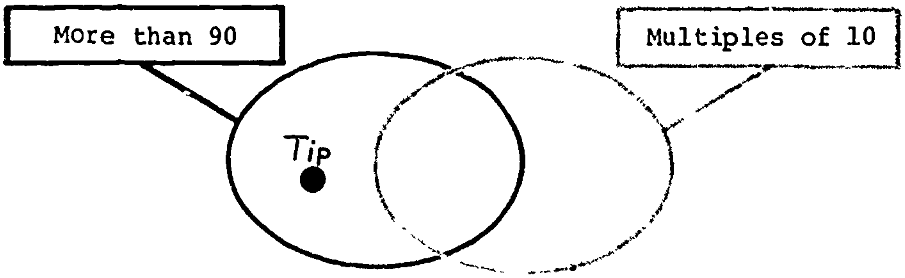
Second Clue



T: Max can be put on this Minicomputer by moving exactly one checker. Which numbers could Max be? (63, 72, 81, 90, 261, 270)

Perhaps a student will observe that the only way to put a multiple of 9 on this Minicomputer is by moving a checker from a white square to a red square.

Third Clue

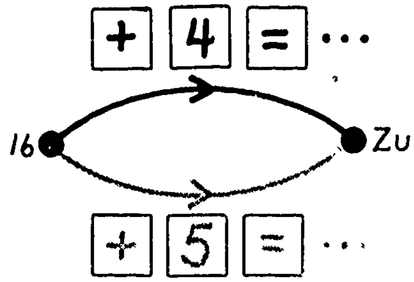


T: What does this string picture tell us about Max? (Max is more than 90 and not a multiple of 10.) Which number is Max? (261)

ACTIVITY A12: WHO IS ZU?

PREREQUISITE: S4, S8, A9

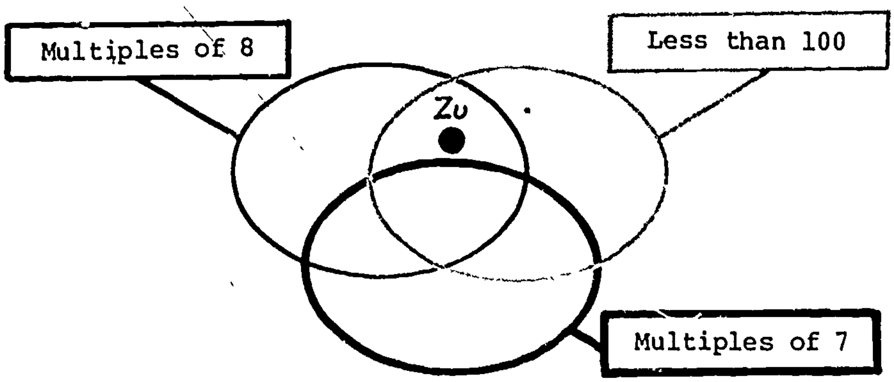
First Clue



T: This arrow picture tells us that if we put 16 on the display of the calculator and press $\boxed{+}\boxed{4}\boxed{=}$ ..., Zu is one of the numbers that will appear. Also if we put 16 on the display and press $\boxed{+}\boxed{5}\boxed{=}$ ..., Zu is one of the numbers that will appear. What are some of the numbers Zu could be?

Let the students work in pairs. Both should put 16 on the display, then one should press $\boxed{+}\boxed{4}\boxed{=}$ ..., and the other $\boxed{+}\boxed{5}\boxed{=}$ ... Encourage them to record the possibilities for Zu. (36, 56, 76, 96, ...) Observe that each possibility is 20 more than the previous possibility.

Second Clue



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T: What does the blue string tell us about Zu? (Zu is less than 100, so Zu is either 36, 56, 76, or 96, ...)

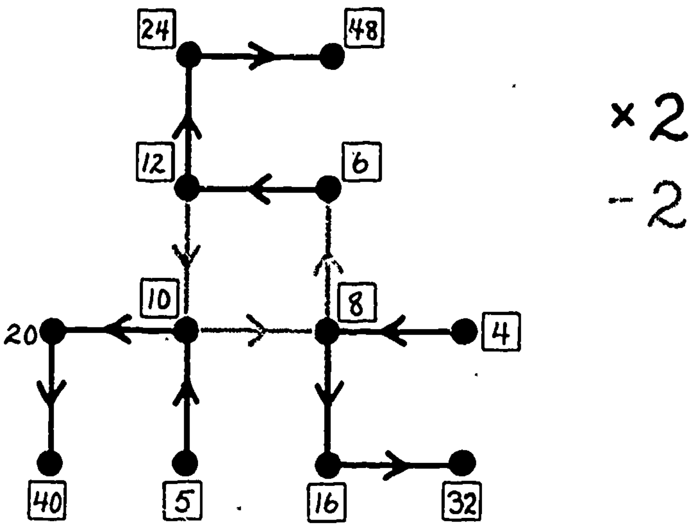
What does the red string tell us about Zu? (Zu is a multiple of 8, so Zu is either 56 or 96.)

What does the green string tell us about Zu? (Zu is not a multiple of 7, so Zu is 96.)

ACTIVITY A13: WHO IS WING?

PREREQUISITE: S9, W2, A1

First clue

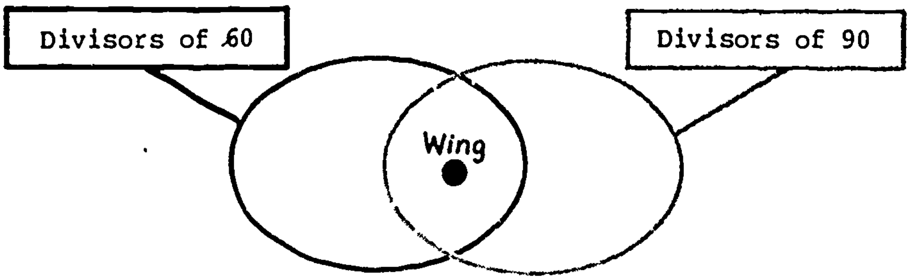


T: Wing is in this arrow picture. Label the dots. Which numbers could Wing be?

Second Clue

T: Wing can be put on the Minicomputer with three checkers all on the same square. Which numbers could Wing be? (6, 12, 24)

Third Clue

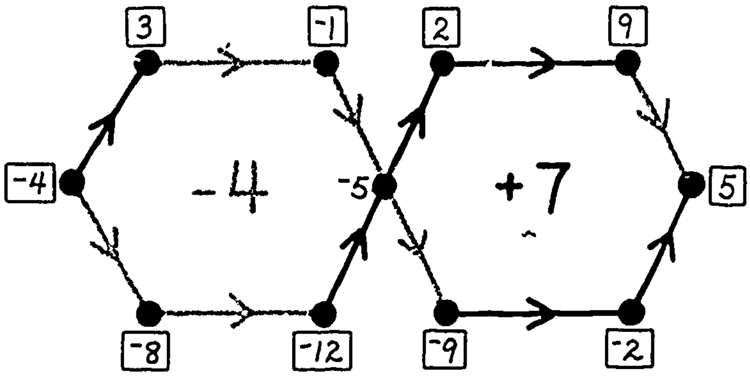


T: What does this string picture tell us about Wing? (Wing is a common divisor of 60 and 90.) Which of these numbers could be Wing? (6)

ACTIVITY A14: WHO IS NAR?

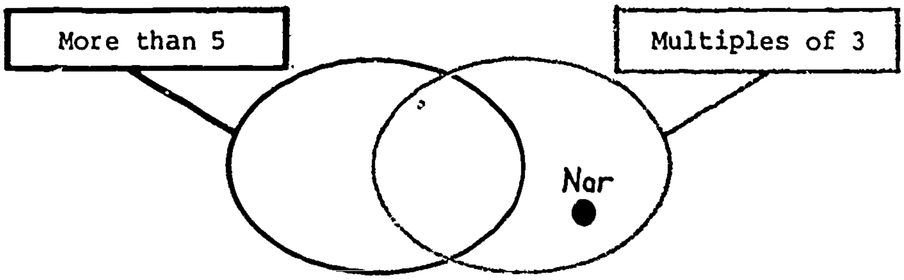
PREREQUISITE: S8, A1, A9

First Clue



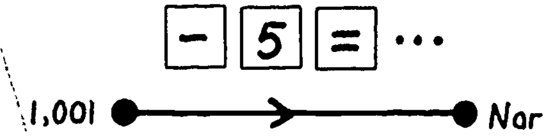
T: Nar is one of the numbers in this arrow picture. Label the dots. Which numbers could Nar be?

Second Clue



T: What does the blue string tell us about Nar? (Nar is a multiple of 3.) Which numbers could be Nar? (9, 3, -9, -12) What does the red string tell us about Nar? (Nar is not more than 5, so we can cross off 9.)

Third Clue



T: Nar is one of the numbers that will appear on the display of the calculator if you start at 1001 and press $\boxed{-}\boxed{5}\boxed{=}$ .... Which numbers could Nar be?

The students may suggest that Nar's ones' digit must be 1 or 6. If necessary, let the students observe the numbers that appear on the display of the calculator.

T: None of these numbers has 1 or 6 for the ones' digit. What is a number close to 0 that will appear on the display of the calculator? (1) Let's see what happens if we start at 1 and press $\boxed{-}\boxed{5}\boxed{=}$ .... (-9 will appear on the display.)

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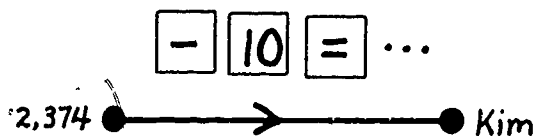
ACTIVITY A15: WHO IS KIM?

PREREQUISITE: N4, A1, A9

First Clue

T: Kim can be put on the ones' board of the Minicomputer with one positive and one negative checker. Which numbers could Kim be? (7, 6, 4, 3, 2, 1, 0, -1, -2, -3, -4, -6, -7)

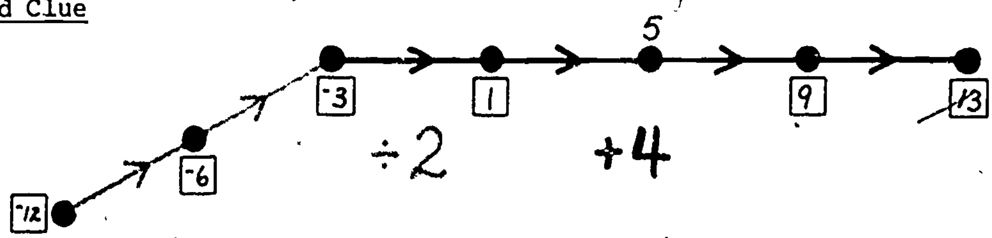
Second Clue



T: What does this arrow picture tell us about Kim?

The students should recognize that Kim could be 4, but they may feel insecure about which negative numbers Kim could be. Ask which number is 4 - 10. (-6) If necessary, let a student do the calculation on a hand-calculator. Conclude that Kim is either 4 or -6.

Third Clue



T: Kim is in this arrow picture. Label the dots. Which number is Kim? (-6)

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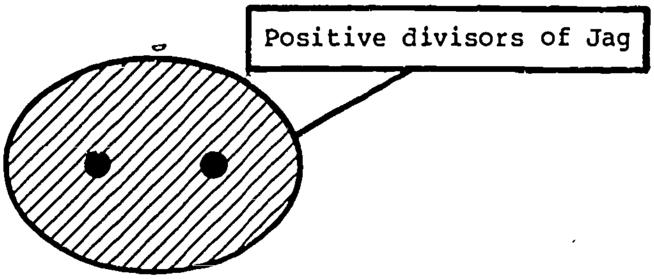
ACTIVITY A16: WHO IS JAG?

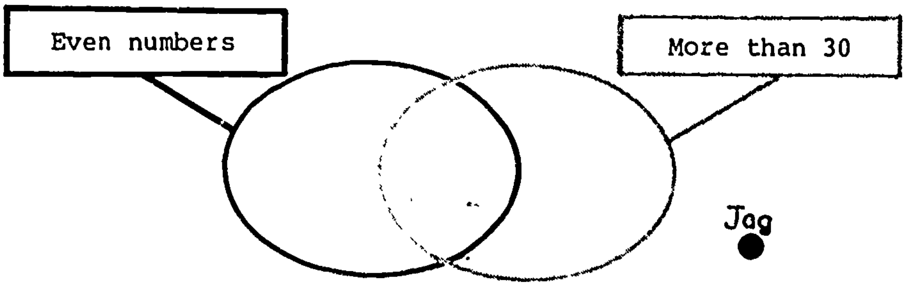
PREREQUISITE: S10, N4, A1

First Clue



T: What does this string picture tell us about Jag? (Jag is a prime number.) Name some numbers Jag could be? (2, 3, 5, 7, 11, 13, ...)

Second Clue

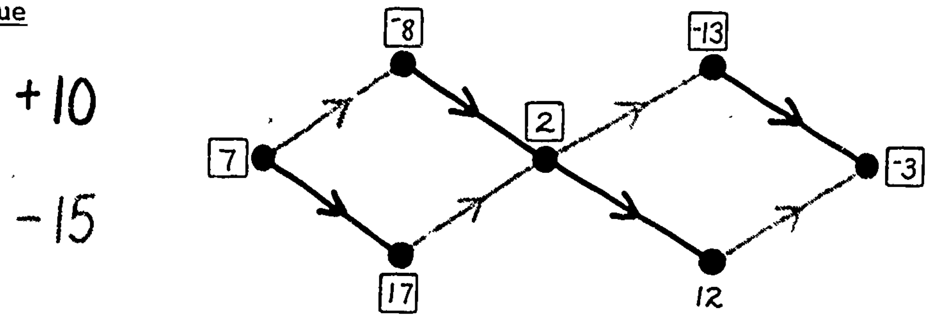


T: What does the red string tell us about Jag? (Jag is an odd number, so we can cross off 2.) What does the blue string tell us about Jag? (Jag is not more than 30.) Which numbers could Jag be? (3, 5, 7, 11, 13, 17, 19, 23, 29)

Third Clue

T: Jag cannot be put on the Minicomputer with exactly two checkers (positive or negative). Which numbers could Jag be? (13, 17, 23, 29)

Fourth Clue

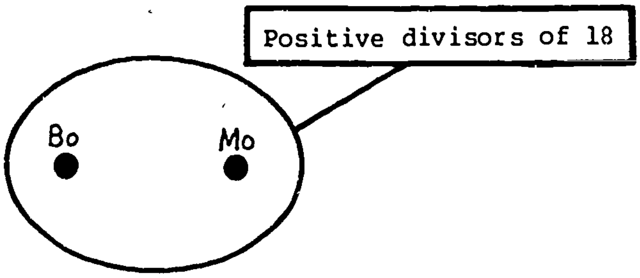


T: Jag is in this arrow picture. Label the dots and determine which number is Jag. (17)

ACTIVITY A17: WHO ARE BO AND MO?

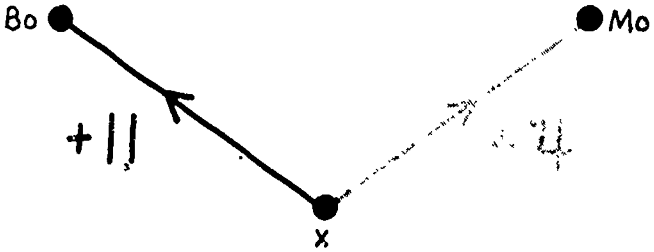
PREREQUISITE: S9, A1

<u>First Clue</u>



T: What does this string picture tell us about Bo and Mo? (Bo and Mo could be 1, 2, 3, 6, 9, or 18.)

<u>Second Clue</u>



Do not write the letter on the board. It is here just to make the description of the lesson easier to follow.

T: What information does this arrow picture give us about Bo and Mo? Which number is larger? (Bo, because Bo is 11 more than <u>this</u> number (pointing to <u>x</u>) and Mo is only 4 more than <u>this</u> number.) Which numbers could be Bo?

Perhaps a student will suggest that Bo is 18. Point to <u>x</u>.

T (pointing to <u>x</u>): If Bo is 18, then this number + 11 is 18. What is this number? (7) If this number (<u>x</u>) is 7, which number is Mo? (11) Could Mo be 11? (No, because 11 is not a divisor of 18.) So Bo cannot be 18. Could Mo be 18? (No, because Mo is smaller than Bo.)

Cross off 18 on the list and ask for another suggestion. Bo is 9 and Mo is 2.

ACTIVITY A18: WHO IS LU?

PREREQUISITE: H6, A1

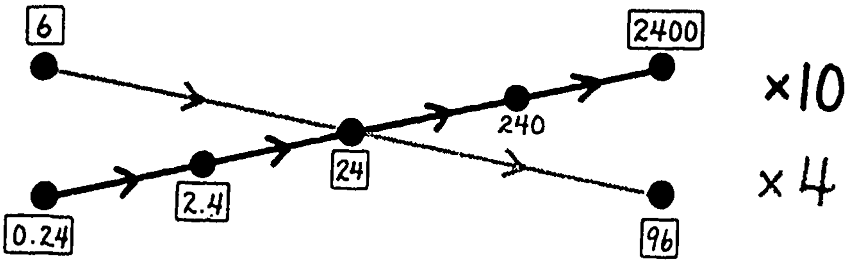
<u>First Clue</u>

9 ☐ 3 ☐ 2 ☐ 10 = Lu

T: One of the symbols +, x, or ÷ is hidden in each blank of this hand-calculator sentence. Each symbol may be used only once. Use your calculator to find the numbers that Lu could be.

Record each hand-calculator sentence on the board as it is suggested. Continue until all six possibilities are listed. Lu could be 2.4, 2.9, 16, 23.5, 50, or 60.

<u>Second Clue</u>



T: Lu is in this arrow picture. Label the dots. Which number is Lu? (2.4)

ERIC

ACTIVITY A19: WHO IS KAMP?

PREREQUISITE: H6, A9

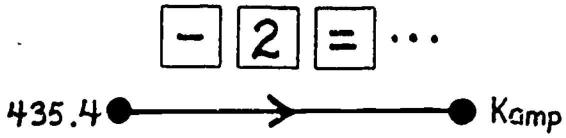
First Clue

$$\boxed{8}\ \boxed{}\ \boxed{2}\ \boxed{}\ \boxed{10}\ \boxed{}\ \boxed{10}\ \boxed{=}\ \text{Kamp}$$

T: One of the symbols + or ÷ is hidden in each blank of this hand-calculator sentence. The same symbol may be used in all three blanks. Use your calculator to determine which numbers Kamp could be.

Record each hand-calculator sentence on the board as it is suggested. Continue until all eight possibilities are listed. Kamp could be 0.04, 0.1, 1.4, 2, 10.4, 11, 24, or 30.

Second Clue



T: Kamp is one of the numbers that will appear on the display of the calculator if we start at 435.4 and press $\boxed{-}\boxed{2}\boxed{=}$ .... What does this clue tell us about Kamp?

Perhaps a student will recognize that Kamp must end in .4 and that the ones' digit must be odd. If necessary, ask the students to put 435.4 on the display and press $\boxed{-}\boxed{2}\boxed{=}$ ... until these patterns are recognized. Conclude and confirm with the calculators that Kamp is 1.4.

ACTIVITY A20: WHO IS LIN?

PREREQUISITE: D3, H6, A9

First Clue

$$\boxed{2}\ \boxed{}\ \boxed{0.5}\ \boxed{}\ \boxed{0.2}\ \boxed{=}\ Lin$$

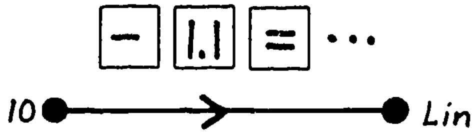
T: One of the symbols +, −, x is hidden in each blank of this hand-calculator sentence. The same symbols may be used in both blanks. Use the calculators to determine which numbers Lin could be.

Record each hand-calculator sentence on the board as it is suggested. Continue until all nine possibilities are listed. Lin could be 2.7, 2.3, 1.7, 1.3, 1.2, 0.8, 0.5, 0.3, or 0.2.

Second Clue

T: Lin can be put on the dimes' board of the Minicomputer with exactly two checkers. Which numbers could Lin be? (0.3, 0.5, 0.8, 1.3)

Third Clue

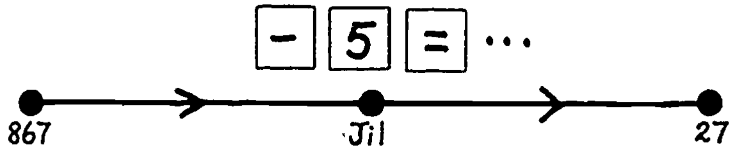


T: Lin is one of the numbers that will appear on the display of the calculator if we start with 10 and press $\boxed{-}\ \boxed{1.1}\ \boxed{=}$ .... Which number is Lin? (1.2)

ACTIVITY A21: WHO IS JIL?

PREREQUISITE: S10, W5, A9

First Clue



T: What does this arrow picture tell us about Jil? (Jil is between 867 and 27, and the ones' digit is either 2 or 7.)

If necessary, ask the students to start with 867 and press $\boxed{-}\boxed{5}\boxed{=}$ ... until the pattern of the ones' digit is recognized.

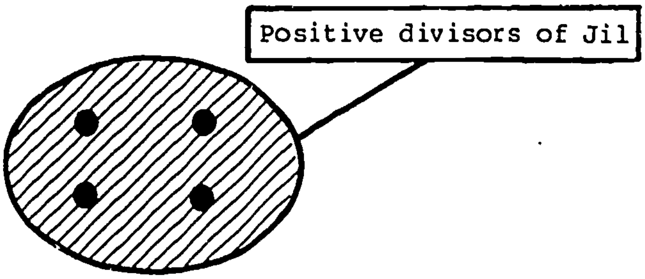
Second Clue

T: Jil can be put on the Minicomputer with one regular (positive) checker on the tens' board and two regular (positive) checkers on the ones' board. Which numbers could Jil be?

Perhaps the students will conclude that Jil could be either 42 or 82. If necessary, tell them that it is possible to put 32 on the Minicomputer with these restrictions. Once they discover how to put 32 on the Minicomputer, configurations for 52 and 92 should also be quickly found. Jil could be 32, 42, 52, 82, or 92.

24.)

Third Clue



T: What does this string picture tell us about Jil? (Jil has exactly four positive divisors.)

Consider each of the possible numbers. It is not necessary to find all the divisors of a number. When the students have determined that a number has five or more divisors, cross it off the list. Jil is 82.

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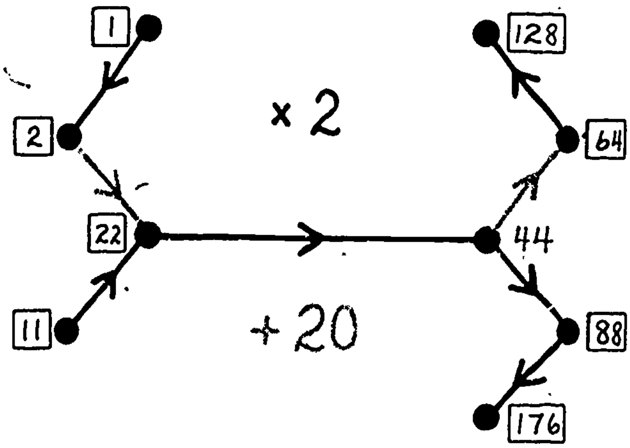
ACTIVITY A22: WHO ARE FLIP AND FLOP?

PREREQUISITE: S8, W5, A1

First Clue



T: Flip and Flop are in this arrow picture. Label the dots and determine which numbers Flip and Flop could be.

Second Clue

T: Flip can be put on the Minicomputer with exactly two checkers on the ones' board and two checkers on the tens' board. Which numbers could Flip be? (22, 44, 64, 88, 128, 176) Flop cannot be put on this Minicomputer with exactly two checkers on the ones' board and two checkers on the tens' board. Which numbers could Flop be? (1, 2, 11)

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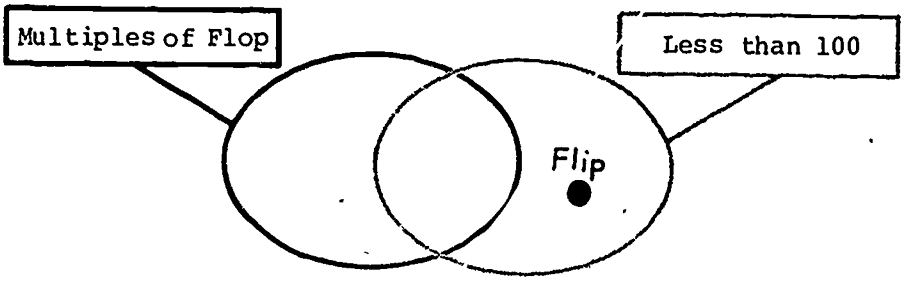
Third Clue



T: What does this string picture tell us about Flip? (Flip is less than 100 and not a multiple of Flop.) Which of these numbers can we cross off our list for Flop? How do you know?

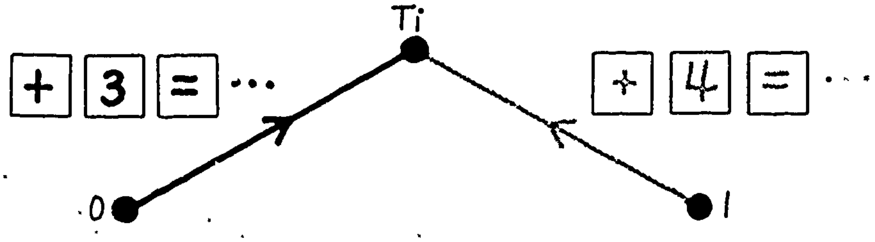
The students should suggest crossing off 128 and 176 and may suggest crossing off other numbers because they are multiples of 1, 2, and 11.

T: Where would Flop be in this string picture? (In the intersection.) How do you know?
Could Flop be 1? (No, because every number is a multiple of 1.)
Could Flop be 2? (No, because all the possibilities for Flip are even numbers.)
Could Flop be 11? (Yes, because 64 is not a multiple of 11.) What numbers are Flip and Flop? (Flip is 64 and Flop is 11.)

ACTIVITY A23: WHO IS TI?

PREREQUISITE: S2, All

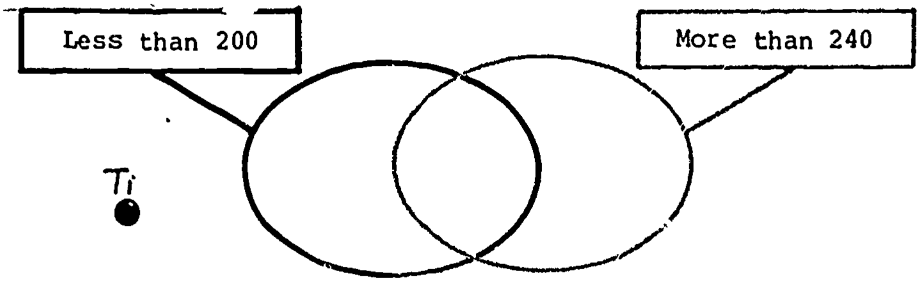
First Clue



T: What does this arrow picture tell us about Ti? (Ti is a multiple of 3 and one more than a multiple of 4.)

Let students work in pairs with one student starting at 0 and pressing $\boxed{+}\boxed{3}\boxed{=}$ ... and the other starting at 1 and pressing $\boxed{+}\boxed{4}\boxed{=}$ .... Ask them to list the numbers that appear on both displays and to try to recognize a pattern. List some of the possibilities (9, 21, 33, 45, 57, ...) on the board. Each number on the list is 12 more than the previous number.

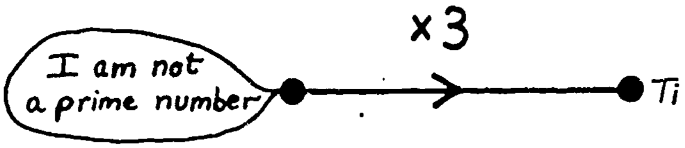
Second Clue



T: What does this string picture tell us about Ti? (Ti is one of the numbers from 200 to 240.)

If necessary, let the students use the calculators to determine the possibilities for Ti. (201, 213, 225, 237)

Third Clue



T: What does this arrow picture tell us about Ti? (Ti ÷ 3 is not a prime number.) What do we need to do to identify Ti? (First we have to divide each number in our list by 3 and then we need to check to see if the new number is a prime.)

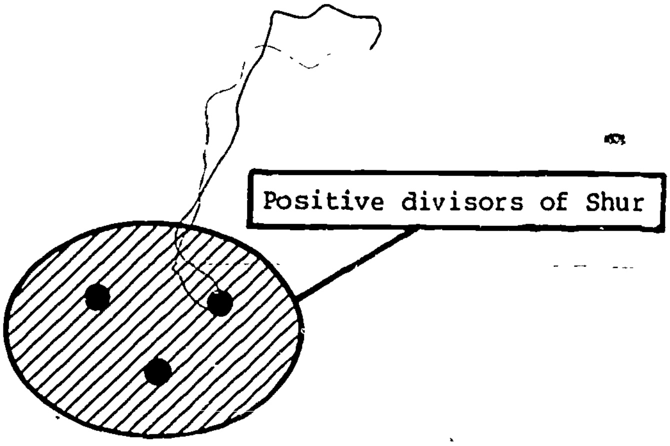
Follow the suggestions of your students. Encourage the use of the hand-calculators. Conclude that Ti is 225 because 3 x 75 = 225 and 75 is not a prime number.

ERIC

ACTIVITY A24: WHO IS SHUR?

PREREQUISITE: S10

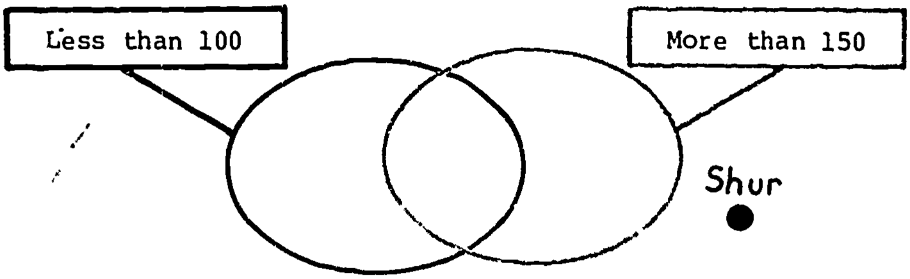
First Clue



T: What does this string picture tell us about Shur? (Shur has exactly three positive divisors so Shur is a perfect square.) Which numbers could Shur be? (4, 9, 25, 49...) Could Shur be 16, since 16 is a perfect square? (No, because 16 has more than three positive divisors.)

Perhaps a student will observe that Shur is the square of a prime number.

Second Clue



T: What does this string picture tell us about Shur? (Shur is one of the numbers from 100 to 150.) Which numbers could Shur be? (121)

It is possible that your students will need to consider each of the perfect squares from 100 to 150 (100, 121, 144) before they can determine that only 121 has exactly three positive divisors.

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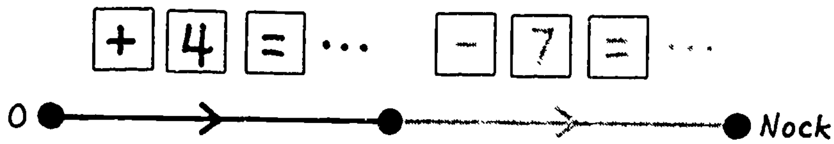
ACTIVITY A25: WHO IS NOCK?

PREREQUISITE: A1, A9

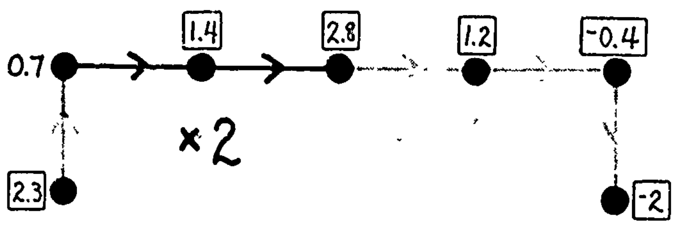
First Clue



T: Nock is one of the numbers that will appear on the display of the calculator if you press $\boxed{+}\boxed{4}\boxed{=}$ ... and then $\boxed{-}\boxed{7}\boxed{=}$ .... Which numbers could Nock be?

Encourage the students to experiment. After a few minutes and many suggestions of possible numbers for Nock, ask if there are any numbers that Nock could not be. As numbers are suggested, challenge the students to put that number of the display. Eventually the class should conclude that Nock can be any integer at all--positive or negative--except 0. Nock cannot be 0 because there is alreadv a dot for 0 in the arrow picture.

Second Clue



T: Nock is in this arrow picture. Label the dots. Which number is Nock? (-2)

Problem Solving With Arrows

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STRAND V: PROBLEM SOLVING WITH ARROWS

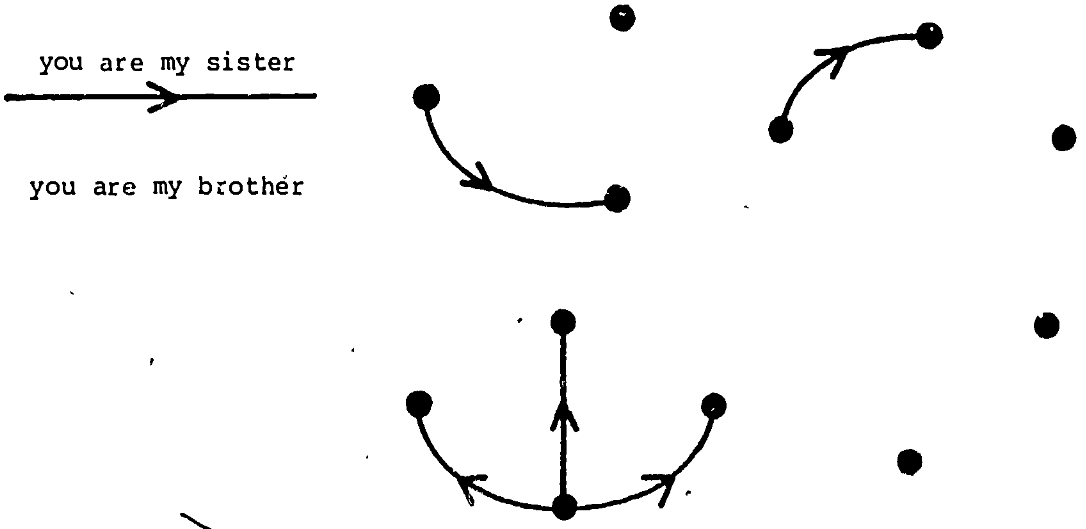
STRAND V: PROBLEM SOLVING WITH ARROWS (Continued)

ERIC

INTRODUCTION

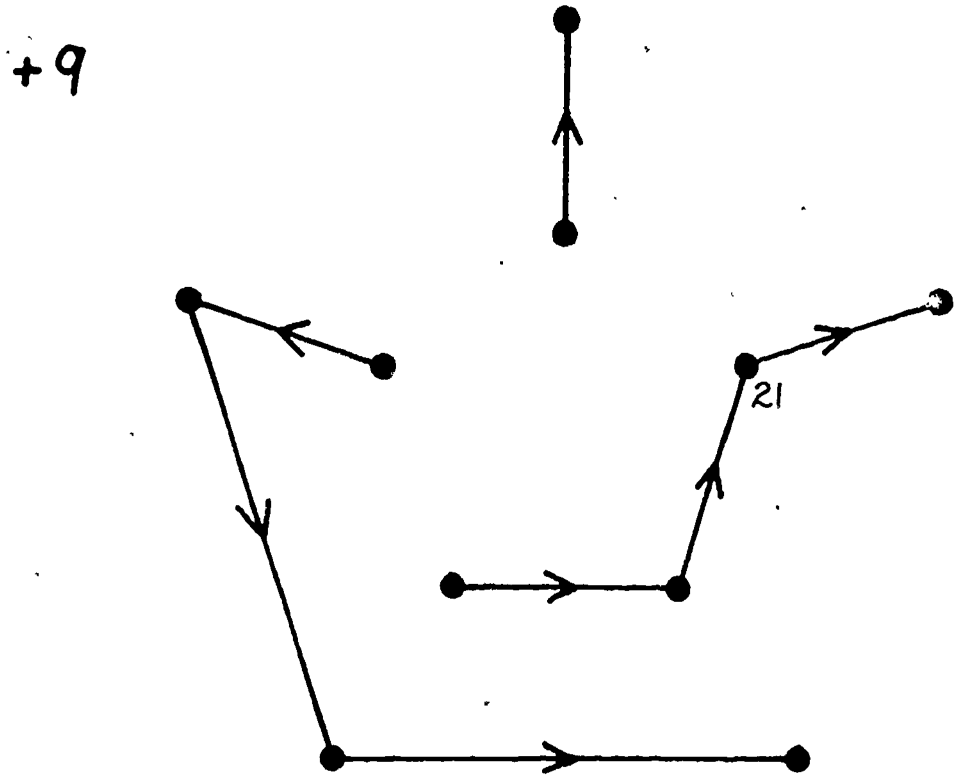
The study of relations among objects is a part of every intellectual pursuit--indeed, at least informally, a part of nearly every human activity. The development of mathematics can be viewed in terms of a series of discoveries of mathematical relations and their exploitation. To mention only a few: Euclid began by formalizing the relations among points and lines; Descartes developed a novel relation between pairs of numbers and points in the plane that provided an impetus for the flowering of the calculus, itself an exploitation of the relations inherent in motion.

Developed with care to avoid esoteric formalism, our use of arrows provides a pictorial language for introducing relations in school mathematics. In the following illustration of sibling relations, dots represent children. Red arrows and blue arrows point from a child to a sister or brother respectively.



To experience the power of the language in posing problems, deduce the sex of some of the children and even determine the location of some arrows missing from the picture (and note any child whose sex cannot be inferred from the information provided).

A natural application of the language of arrows lies in representing numerical relations. In the following illustration, a red arrow signifies the function "add nine" and a blue arrow signifies "subtract six". With one number of the series identified, students are virtually compelled to identify the remaining numbers. Calculation skills are exercised through a direct, dynamic involvement with arithmetic.



Arrows are used to develop the concepts of arithmetic and they offer many occasions for computational exercise. At no time is facility with the mechanism of the language an objective--rather the development of related reasoning and computational abilities is foremost. Even young or weak students can work with the arrow diagram of a relation before being able to discuss the same situation verbally. The language frees students from the constraints of an undeveloped verbal ability.

The danger of using arrow diagrams as simply another format for arithmetic drill requires careful attention to the development of situations. Both non-numerical relations and numerical relations are presented in the activities of this strand. In either case, situations must be thought provoking and challenging.

When presenting the activities, draw large arrow pictures with long, graceful arrows and plan the layout of the arrows in advance so that the significance of the arrow picture will be easy to interpret. Unless otherwise indicated, all the arrows of the same color in an arrow picture represent the same relation. Red arrows in the illustrations are indicated by a thin black line, blue by light gray, green by thick black, and orange by dark gray.

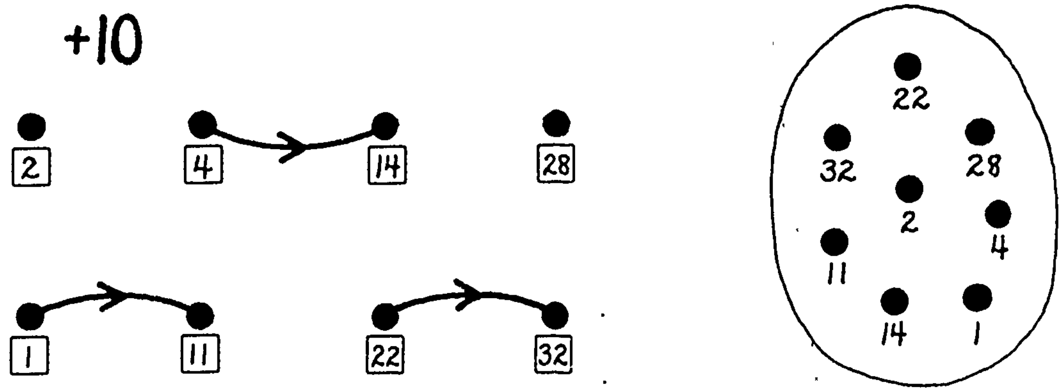
When tracing an arrow, point with your left hand to the starting dot and trace the arrow with your right hand, "reading" the arrow as you do so. For example, when tracing a x5 arrow from 10 to 50, say "Ten ...times five... is fifty".

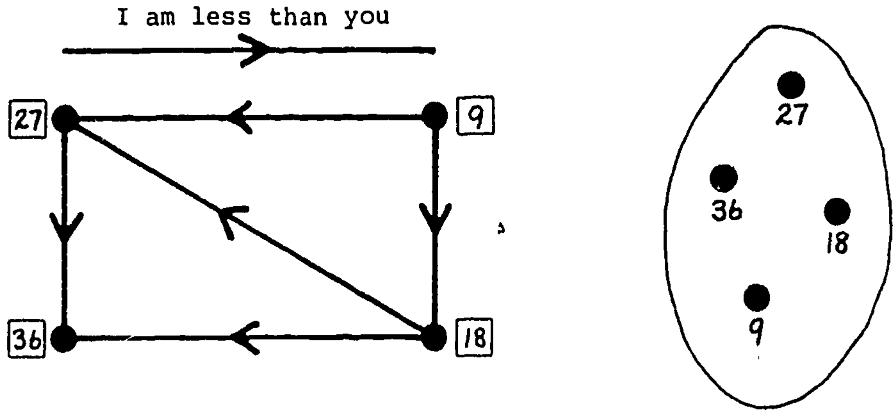
ACTIVITY B1: FISHING FOR NUMBERS

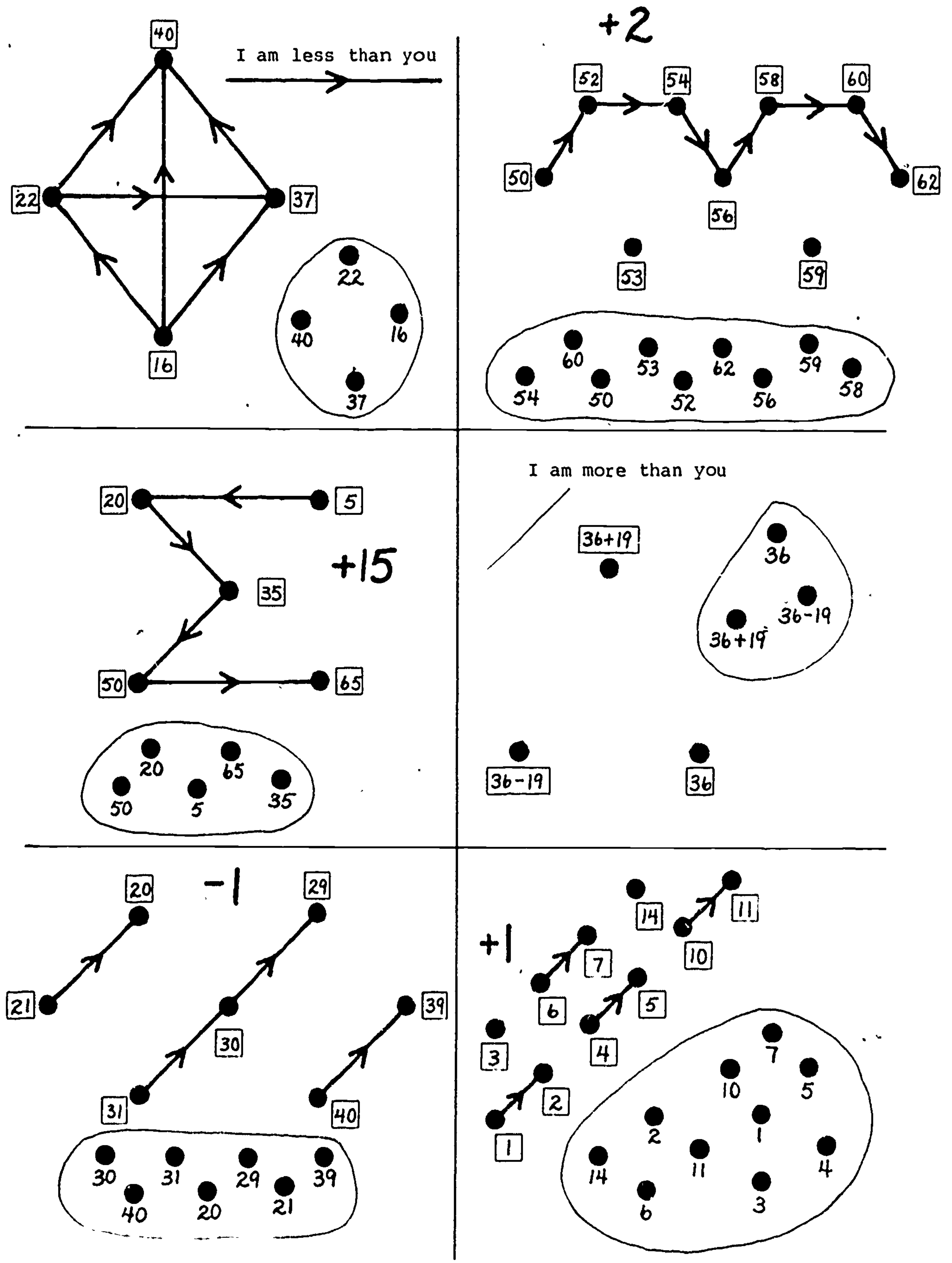
PREREQUISITE: None

OBJECTIVE: Using given sets of numbers, students will label a variety of arrow pictures.

After drawing one of the following pictures on the board, ask the class to place the numbers in the string in their proper places in the arrow picture. The answers are shown in boxes. Encourage students to discuss various strategies for placing numbers. Additional problems of this kind follow. Evidently, one can construct a great variety of such problems, depending on the nature and needs of the class. In particular, one or more problems of this kind can be used in the odd five minutes.





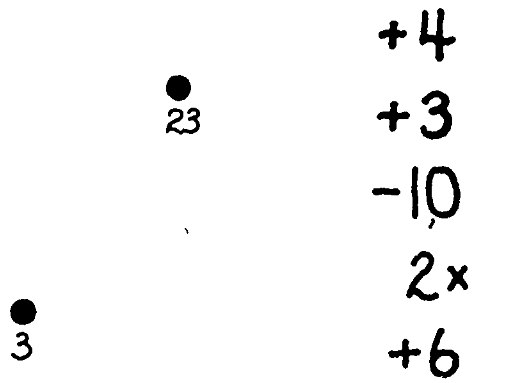

I am less than you
40
22
37
16
22
40
16
37
+2
52
54
58
60
50
56
62
53
59
60
53
62
59
54
50
52
56
58
20
5
+15
35
50
65
20
65
50
5
35
I am more than you
36+19
36
36-19
36+19
36-19
36
-1
20
29
21
30
39
31
40
30
31
29
39
40
20
21
14
11
+1
7
10
6
5
3
4
2
1
7
10
5
2
1
14
11
4
6
3

ACTIVITY B2: WHICH ROAD?

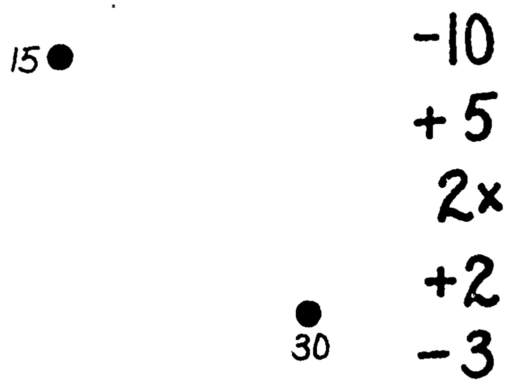
PREREQUISITE: None

OBJECTIVE: Students will investigate and decide which of five functions could be used to build a road between two given numbers if only one kind of arrow is used throughout the road.

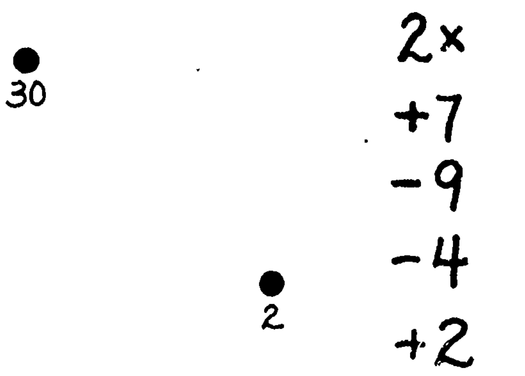
Provide two numbers and a list of operations. Ask students to build a road between the two numbers using the same kind of arrow throughout the road. Students must choose among the operations listed, some of which may not work. After the students choose one workable operation, encourage them to look for a second or third solution. Of course, you can construct many more problems of this type. Begin with one of the following:



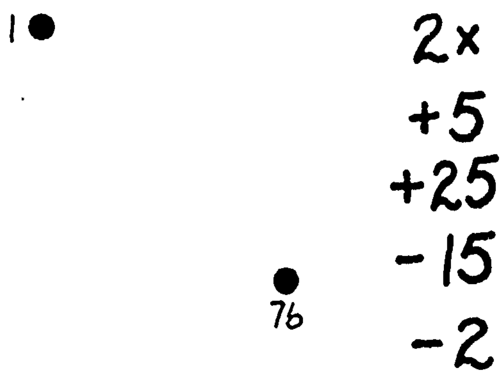
(Solutions: -10 or +4)



(Solutions: 2x, -3, or +5)



(Solutions: +7, -4, or +2)



(Solutions: +25, +5, or -15)

18 ●

● 82

-40
+8
2x
+16
+32

(Solutions: +32, +8, or +16)

750 ●

● 250

+300
-250
+100
3x
+200

(Solutions: 3x, -250, or +100)

● 36

● 6

2x
3x
+15
6x
-6

(Solutions: 6x, +15, or -6)

● 80

● 16

2x
+8
-16
5x
+34

(Solutions: 5x, +8, or -16)

5 ●

● 125

3x
+25
5x
-20
+60

(Solutions: 5x, +60, or -20)

57 ●

● 19

+2
4x
3x
-19
+39

(Solutions: 3x, +2, or -19)

ACTIVITY B3: BUILDING ROADS

PREREQUISITE: None

OBJECTIVE: Students will build arrow roads using two types of arrows.

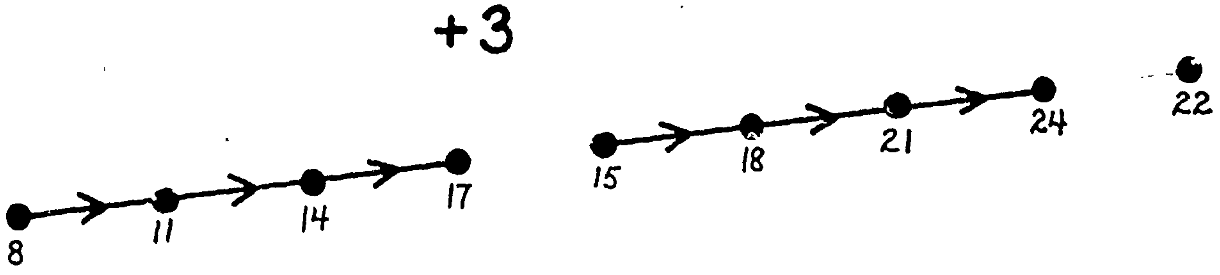
All of the problems in this activity have the same format: Given two numbers and two operations depicted by arrows, build an arrow road from one number to the other number. This format allows great flexibility. The operations to be used determine the mathematical content and difficulty of the activity. Altering the starting number and ending number affects the type of numbers (whole, negative, decimal) involved and also can increase the difficulty of the problem.

Most problems that involve building arrow roads have more than one solution. This fact encourages students to explore the situation as they seek their own solutions. To encourage this exploration, allow time for many students to build roads successfully and record several solutions on the chalkboard for each problem. Emphasize neither the first solution nor the shortest solutions.

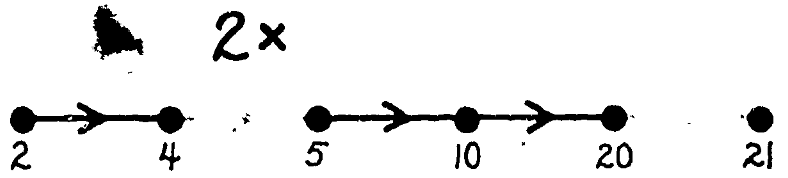
We suggest that your class solve one or two problems collectively at the board and then attempt additional problems individually. After sufficient individual worktime, record student solutions on the board.

Examples with one solution shown:

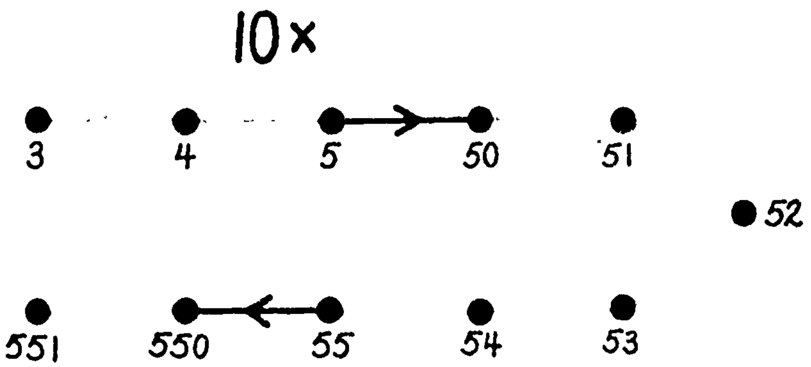
1. Build a road from 8 to 22 using +3 and -2 arrows.



2. Build a road from 2 to 21 using 2x and +1 arrows.



3. Build a road from 3 to 52; then continue the road from 52 to 551 using 10x and +1 arrows.



Here are several of the many, many possible situations for your students to work with. Build a road:

from 17 to 30 using +1 and +2 arrows;
from 8 to 19 using +3 and −2 arrows;
from 10 to 10 using +3 and −2 arrows;
from 0 to 0 using −4 and +3 arrows;
from 61 to 61 using −4 and +3 arrows;
from 5 to 125 using 2x and +3 arrows;
from 0 to 312 using 10x and +1 arrows;
from 100 to 1000 using 2x and +5 arrows;
from 0 to 451 using 10x and +1 arrows.

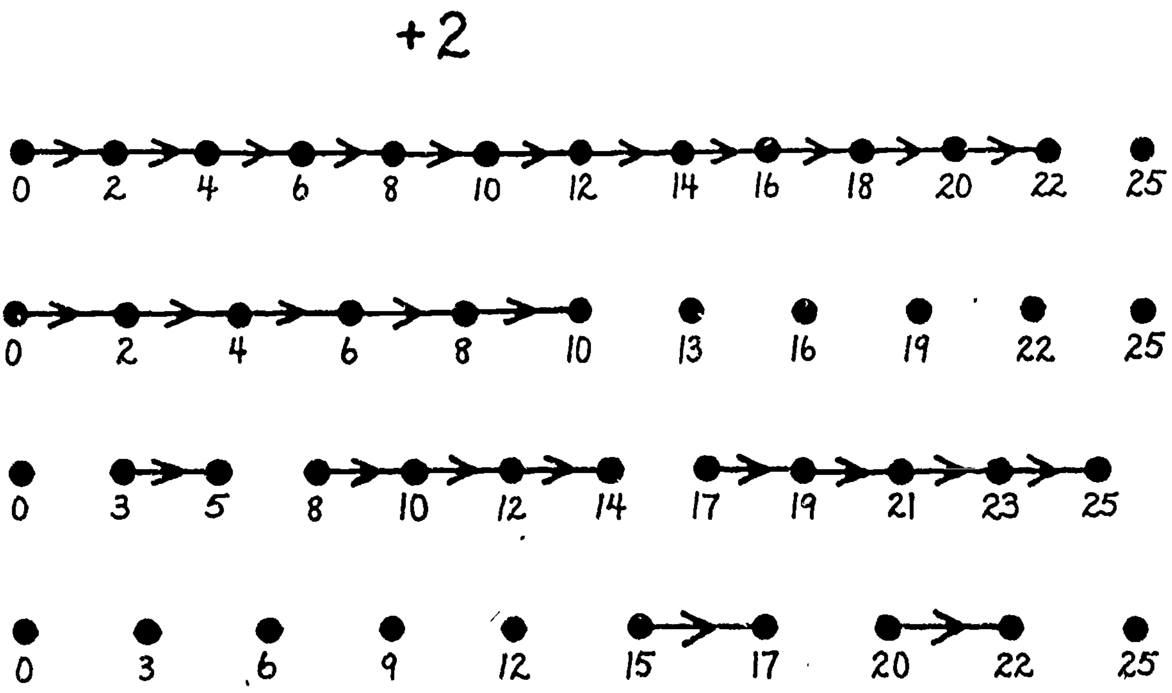
ACTIVITY B4: BUILDING ROADS, STORY #1

PREREQUISITE: Activity B3

OBJECTIVE: Using arrow pictures, students will find ways to put postage on a letter using only specified denominations of stamps.

T: Lori has a letter that needs 25¢ postage. She could not find a 25¢ stamp, but she did find several 2¢ stamps and 3¢ stamps. Can Lori put 25¢ postage on her letter using 2¢ and 3¢ stamps?

Encourage students to use arrow pictures to solve the problem and illustrate their solutions on the board. Here are several solutions.



One can construct similar problems with ease. For example, use 3¢ stamps, 5¢ stamps, and 7¢ stamps for a package needing 50¢ postage. Find more than one solution.

ACTIVITY B5: BUILDING ROADS, STORY #2

PREREQUISITE: Activities B3 and N2

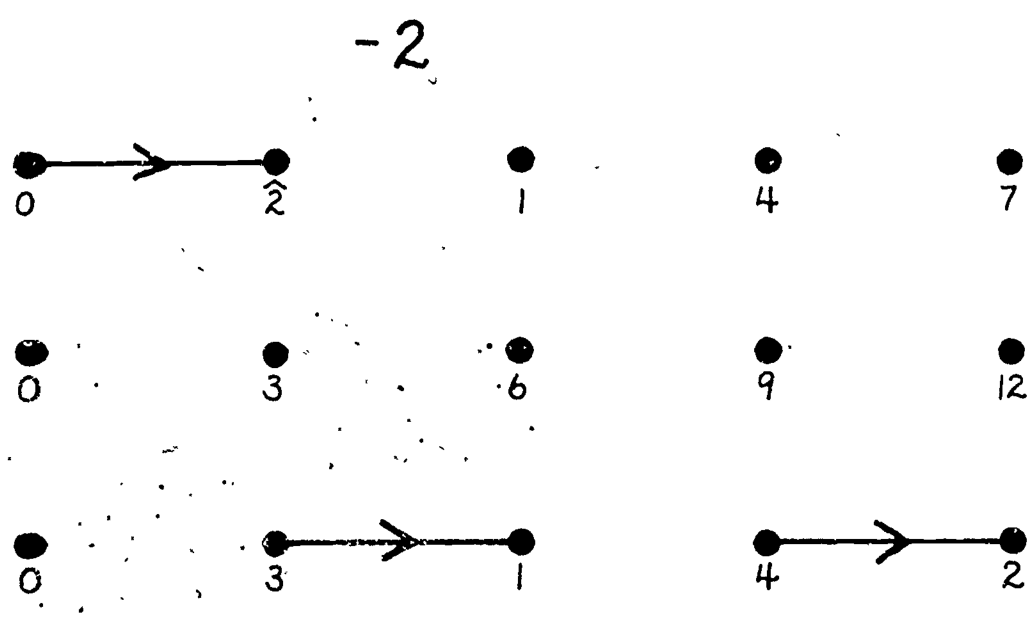
OBJECTIVE: Using arrow pictures, students will decide Pedro's winnings after playing several games of checkers.

T: Pedro likes to play checkers with his grandmother. Each time Pedro wins, his grandmother gives him 3¢. Each time his grandmother wins, Pedro gives her 2¢. Record this information on the board.

Pedro wins: Pedro gets 3¢
Grandmother wins: Pedro gives up 2¢

T: Suppose that Pedro starts with no money. What could happen in four games? Draw an arrow picture to show Pedro's possible wins and losses.

Some possibilities are as follows:

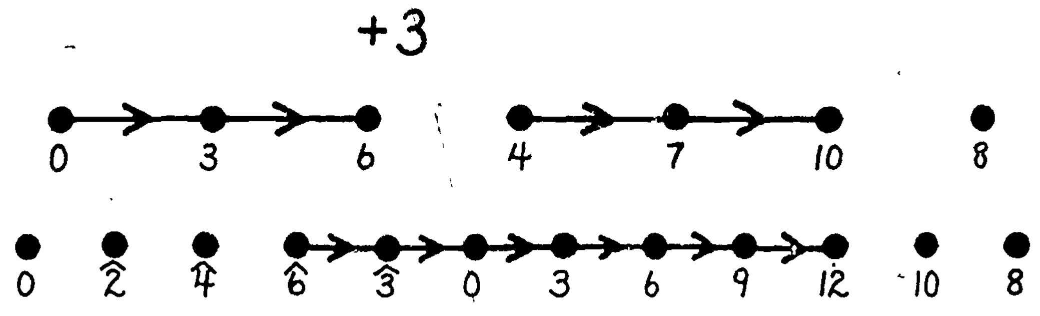


Record information in a table as follows, then encourage the class to look for patterns in the last column. Ask what difference, if any, the order of red and blue arrows makes in the result for Pedro. Extend the discussion if you wish by considering more than four games.

Number of games Pedro wins	Number of games Grandmother wins	Pedro's winnings
4	0	12¢
3	1	7¢
2	2	2¢
1	3	-3¢
0	4	-8¢

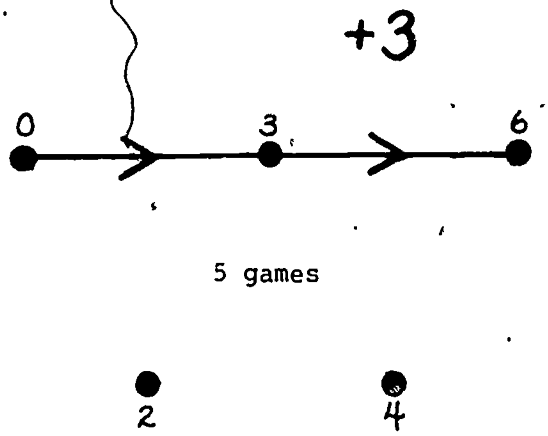
T: One afternoon Pedro and his grandmother played several games of checkers. When they finished, Pedro had won exactly 8¢. Is this possible? (Yes) Draw an arrow picture to show how Pedro could win 8¢.

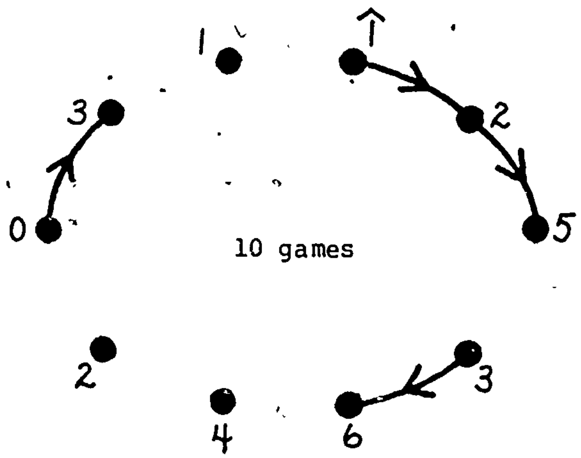
Here are two of an infinite number of possible solutions. Encourage discussion about the number of possibilities.

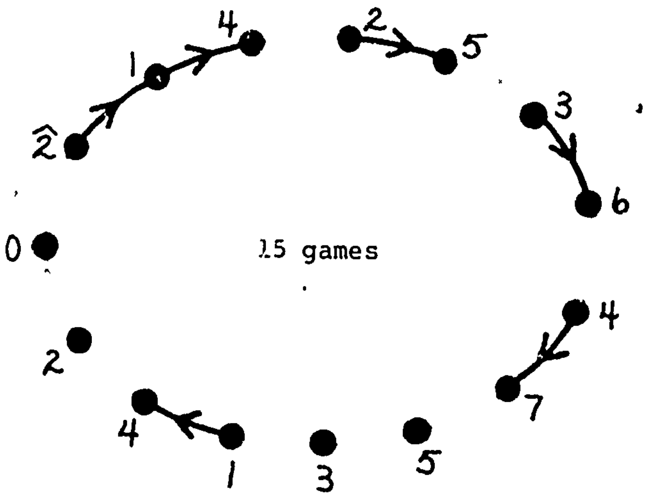


T: One day after several games, Pedro and his grandmother each had won as much money as each had lost; so Pedro was back to 0¢. How could this have happened?

There are many solutions. In the first one depicted below, Pedro wins twice and loses three times. Repeating this, they can break even after every fifth game.







ACTIVITY B6: ARROW ROADS

PREREQUISITE: None

OBJECTIVE: Students will investigate which numbers can be at the end of roads that use two kinds of arrows.

Exercise 1

T: Today we will use two kinds of arrows. The red arrows will represent x10 and the blue arrows will represent +1. What are the possible ending numbers of a road that starts at 0 and uses exactly two red arrows and ten blue arrows?

Let students work independently for awhile and then ask students to put some roads on the board. There are many possible ending numbers and a chart of those found will help the class to look for patterns among them. Ask students to determine the largest ending number and the smallest ending number among those found. One observation that might be made is that all solutions except 1,000 (if it is found) have three digits (this is obvious when we know that the smallest solution is 109 and the largest is 1,000). Another observation--and one that leads to other ending numbers--is that the sum of the digits (except for 1,000) is always 10. With this observation, students can propose three digit numbers whose digits add to 10 and try to find arrow roads that lead to them. Determining the complete lis. f solutions might make an interesting project. They are listed on the following page.

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1000	820	730	640	550	460	370	280	190
910	811	721	631	541	451	361	271	181
901	802	712	622	532	442	352	262	172
		703	613	523	433	343	253	163
			604	514	424	334	244	154
				505	415	325	235	145
					406	316	226	136
						307	217	127
							208	118
								109

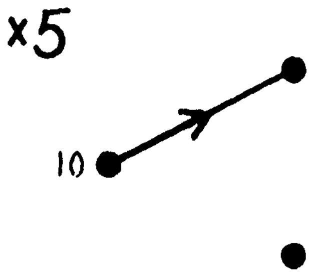
Exercise 2

T: Riz is the ending number of a red-blue road starting at 10, using exactly two red arrows and exactly two blue arrows.

Ask students to draw all possible arrow roads beginning at 10, using two red arrows and two blue arrows where the red arrows are for x5 and the blue arrows are for -2.

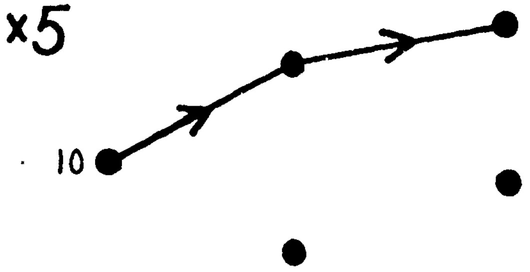
Allow the class to work independently for awhile before initiating a discussion of systematic procedure. Gradually build a tree of arrows in which arrow roads are embedded as shown below.

Begin with a dot labeled "10" and start both a red and a blue arrow.

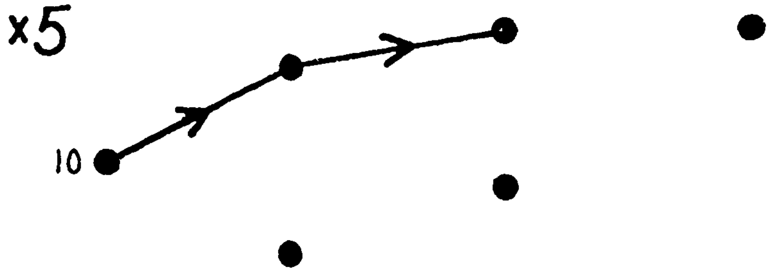


ERIC Full Text Provided by ERIC

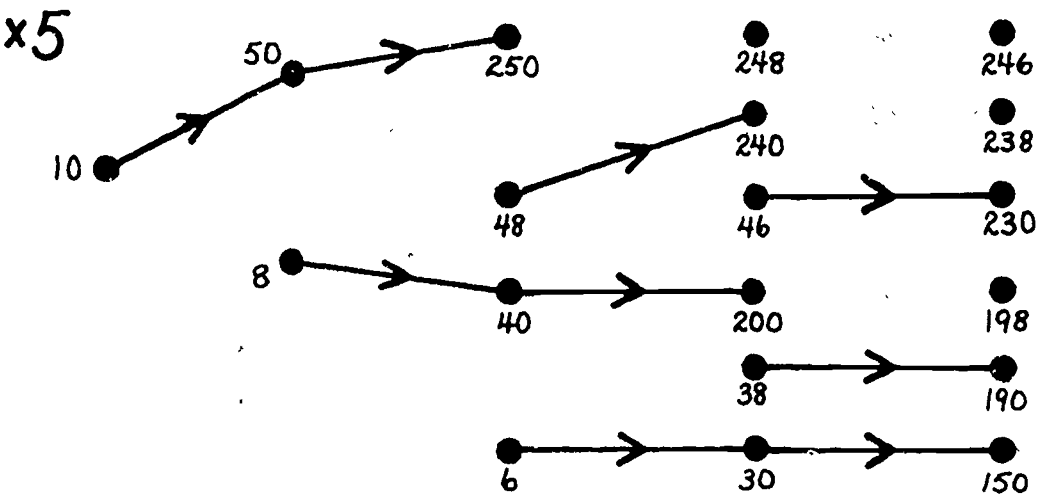
T: If we choose the red arrow first, then what are the choices for the second arrow? (A red or a blue arrow.)



T: If we choose the red-red road, then what are the choices for the third arrow? (It must be a blue arrow.)



Continue in this same manner o discuss the choices and to extend your arrow picture. Remind the class that the roads are limited to two red and two blue arrows. Have the students label the dots to find all of the numbers at the end of the arrow roads. Your completed picture should look something like this.



You can easily construct similar problems that meet the needs and interests of your class. Choose two kinds of arrows and a starting number. Examples follow.

1. Deca is the ending number of a red-blue arrow road starting at 3, using one red arrow and three blue arrows, where the red arrows represent x10 and the blue arrows represent +10. (60, 150, 240, and 330)

2. Chex is the ending number of a red-blue arrow road starting at 3, using one red arrow and two blue arrows, where the red arrows represent x4 and the blue arrows represent +5. (22, 37, and 52)

3. Tex is the ending number of a red-blue arrow road starting at 10, using two red arrows and three blue arrows, where the red arrows represent x5 and the blue arrows represent +1. (325, 305, 301, 285, 281, 277, 265, 261, 257, and 253)

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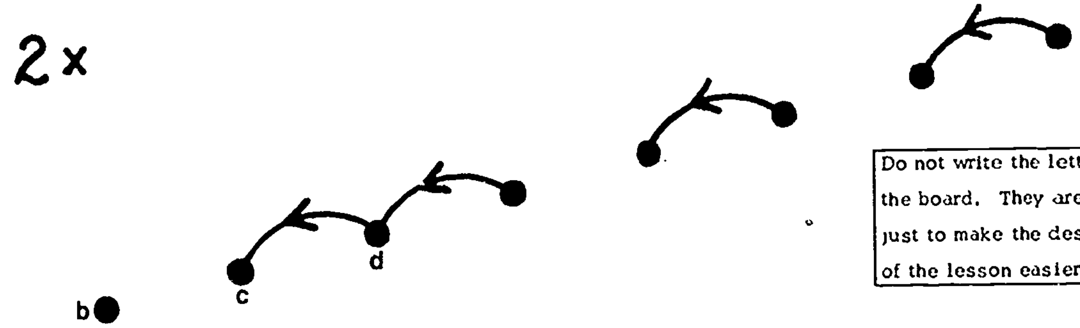
ACTIVITY B7: RETURN ARROWS

PREREQUISITE: None

OBJECTIVE: Students will use return arrows to label dots in arrow pictures and to discover the identity of a secret number.

Exercise 1

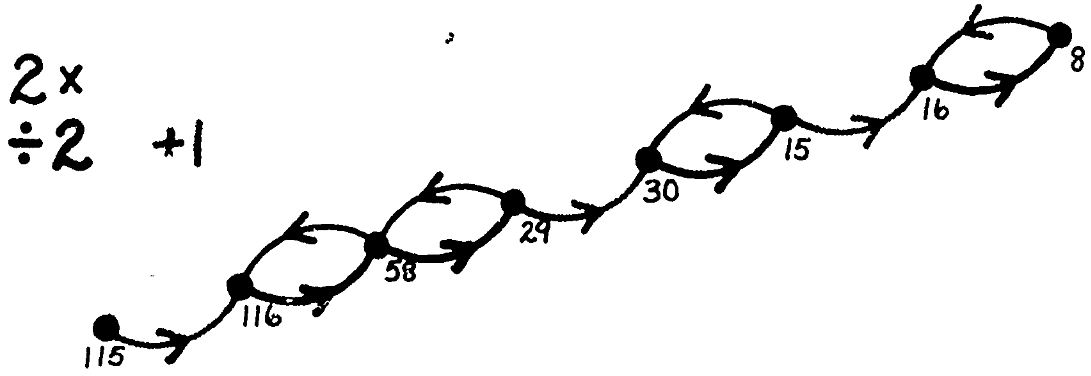
Draw this picture on the board.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: Where is the largest number in this arrow picture? How do you know? Where is the smallest number? How do you know?

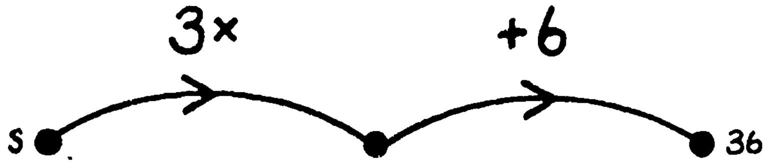
Avoid reaching any conclusions at this time. Label b "115". Ask what number c is. Draw an orange arrow from b to c. Ask what the orange arrow could represent. (+1) Continue by drawing a green arrow from c to d. (1/2 x or ÷2) Complete drawing the return arrows and labeling the dots. Your completed picture should be similar to the following picture.



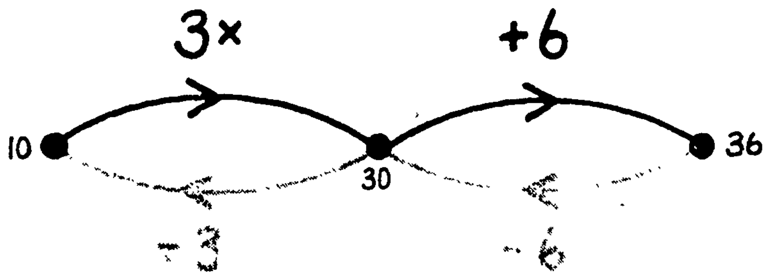
Let students identify the largest and smallest numbers in the picture.

Exercise 2

Draw this picture on the board.

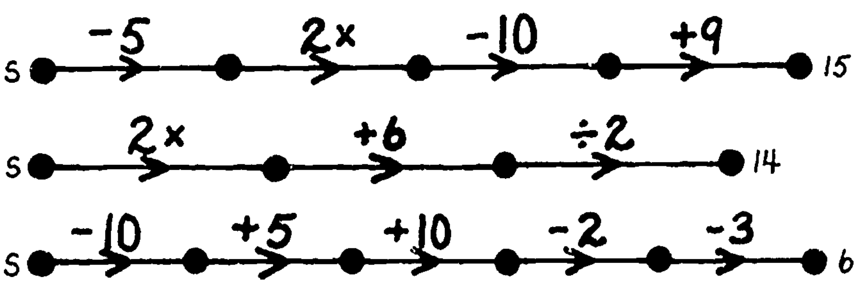


T: S is a secret number. If we start at S and follow a 3x and then a +6 arrow, the ending number is 36. What is the secret number? Let's use return arrows to help us solve the problem.



NOTE: Problems such as this provide an introduction to algebraic thinking insofar as they are a representation of the linear equations that we study so diligently in algebra classes. The problem above is to find the "unknown", given that 3S + 6 = 36. Picturing this equation as an arrow road with the implicit return arrows helps us to organize our thinking in trying to "solve for S".

Many similar problems are available. Check any algebra text or construct your own. Of course, we are not limited to two-step roads in these problems. In fact, when the conventions are well-understood, longer roads are more interesting. Note that in the last of the following examples, the starting number and the ending number are the same.



(S = 13)

(S = 11)

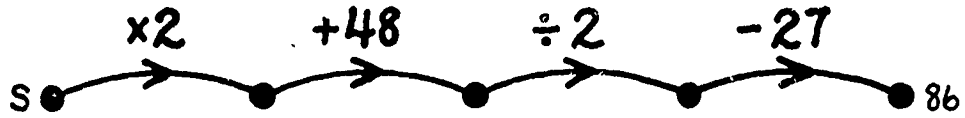
(S = 6)

Exercise 3

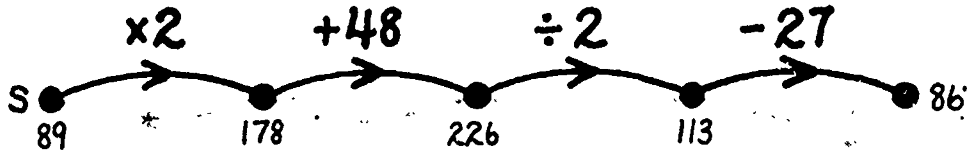
Distribute hand-calculators.

T: S is a secret number on my hand-calculator. If I perform these operations on my hand-calculator in sequence: x2, +48, ÷2, and −27; then press $\boxed{=}$, my display will read 86. What is my secret number?

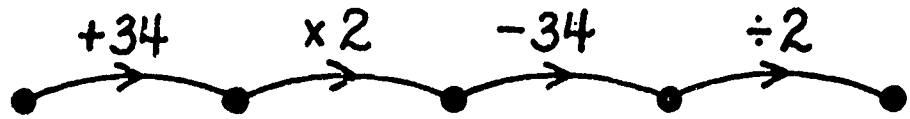
Draw this arrow picture on the board.

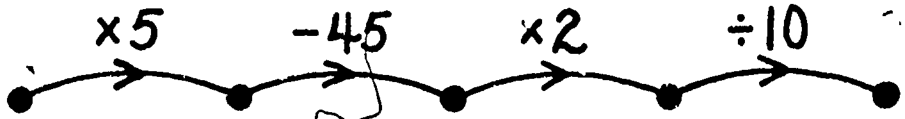


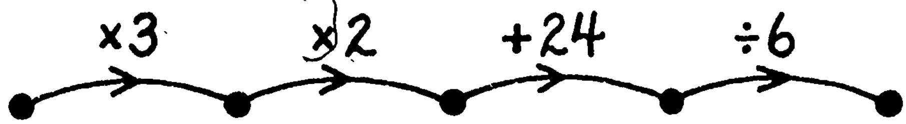
Trace the return arrows and label the dots, using the hand-calculators if necessary. Conclude that S is 89.



Repeat the exercise with other sequences of operations. Let the students take turns choosing the secret number.







ACTIVITY B8: FAMILY RELATIONS

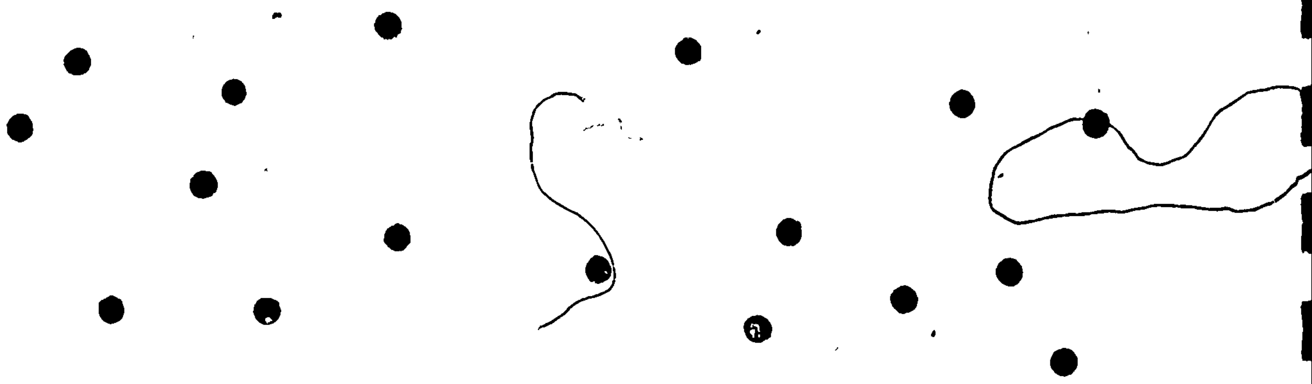
PREREQUISITE: None

OBJECTIVE: Given arrows representing non-numerical relations, students will label the dots and draw additional arrows.

Exercise 1: Brother-Sister Relationships

Draw this arrow picture.

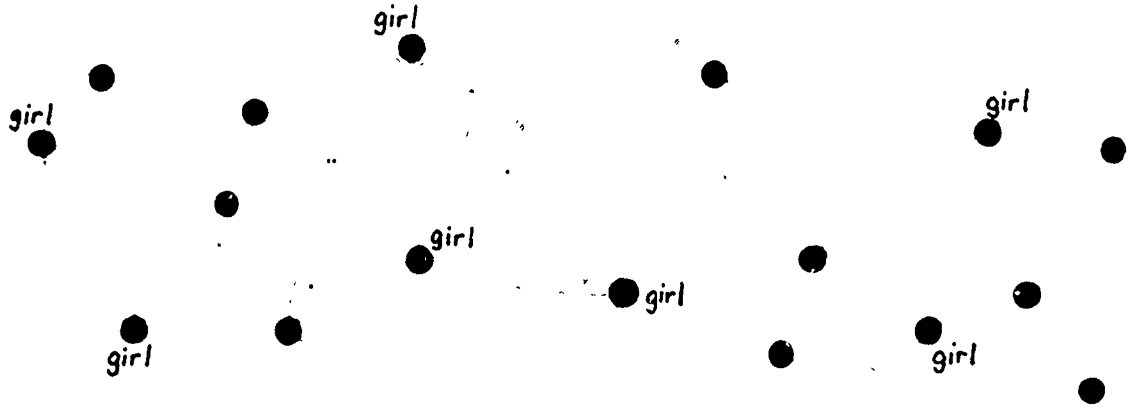
you are my sister



T: The dots represent children on a playground. Some are pointing to their brothers or sisters. Which dots are for girls? Which dots cannot be labeled? Some blue arrows are missing. Where can we be certain that another blue arrow can be drawn in this picture?

When a dot has no arrow pointing toward it, we cannot determine whether the child is a boy or a girl. A further clue is needed. The next illustration shows all of the labels and arrows that can be added without more information.

you are my sister

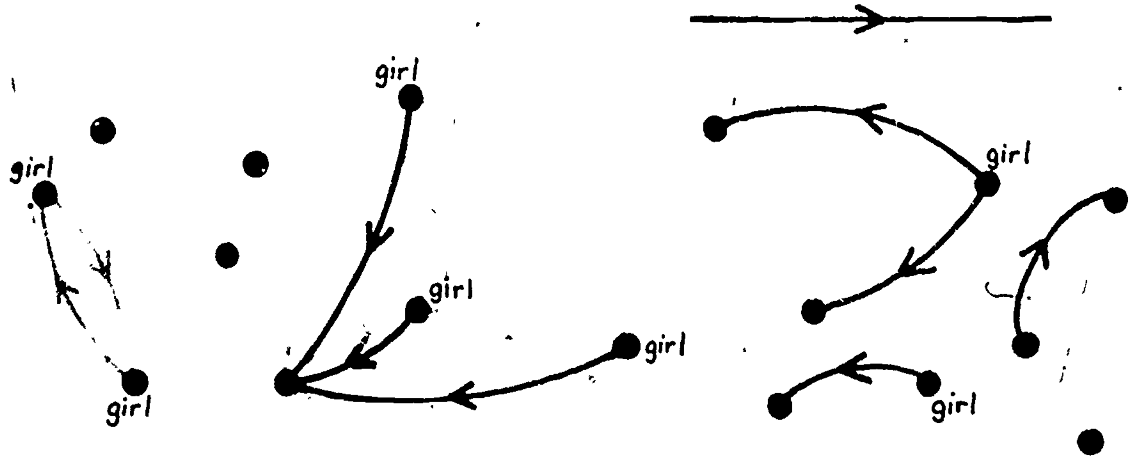


T: No more blue arrows can be drawn in this picture. But here are a few red arrows that are for "you are my brother".

Add red arrows to your picture as shown below.

you are my sister

you are my brother



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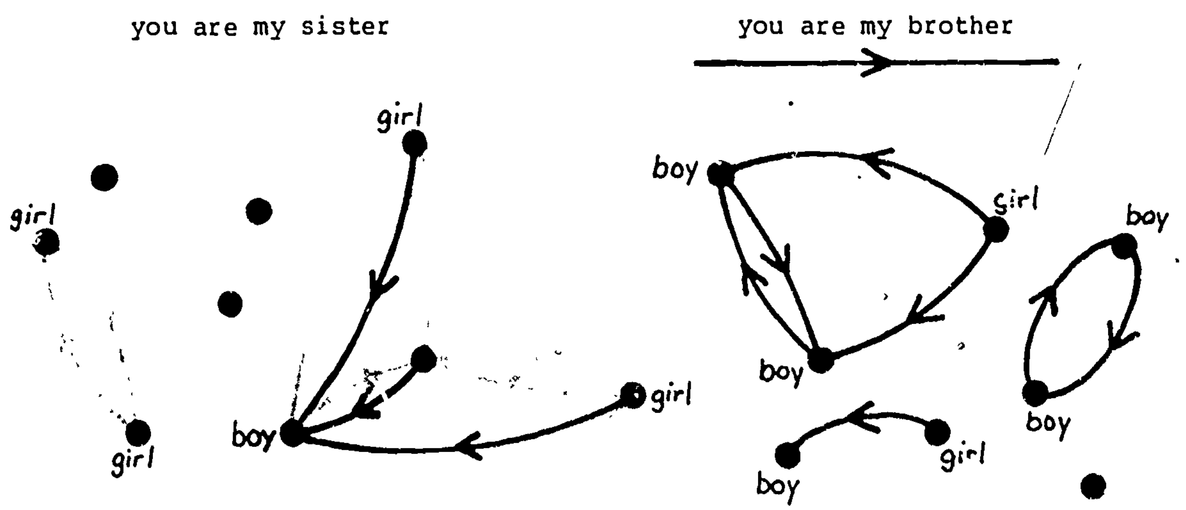
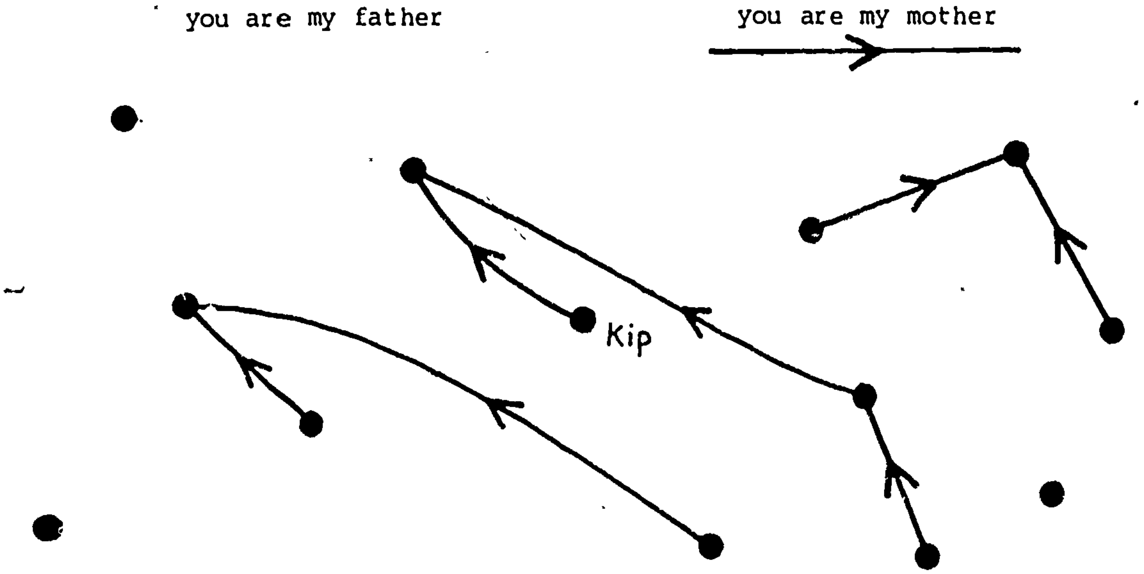
Invite students to label dots and to draw more red arrows when possible. A complete drawing is shown here.

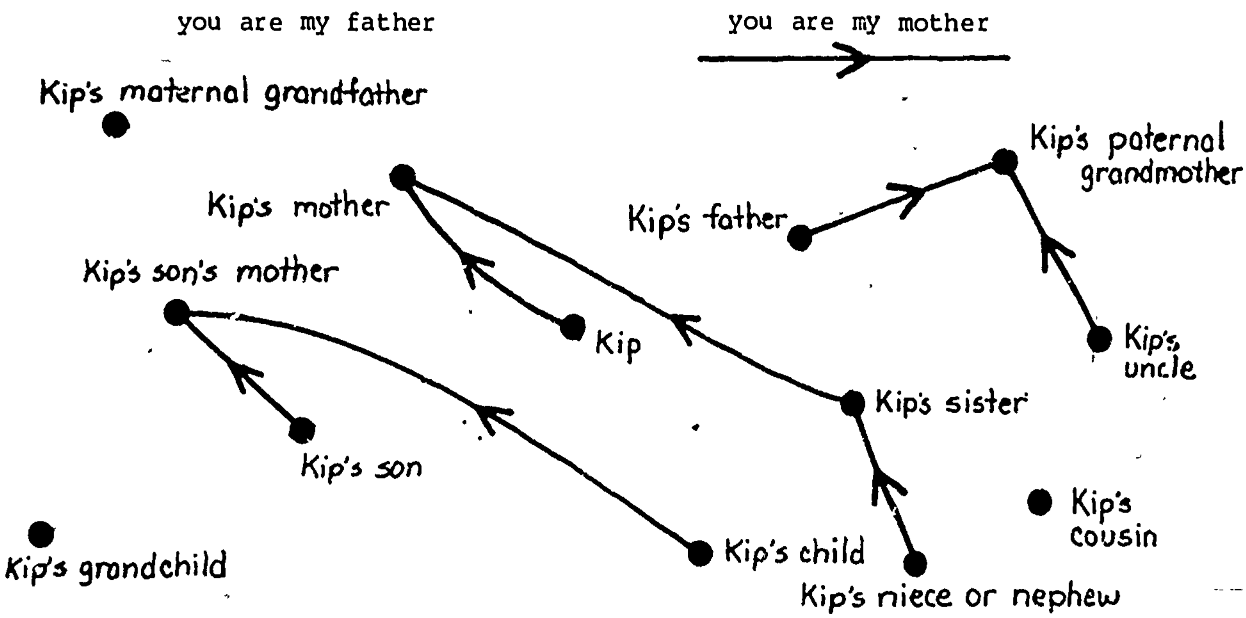


Exercise 2: A Family Reunion

Draw the following arrow picture.



Explain that the dots show Kip's relatives who gathered for a family reunion. A red arrow points to the person's mother while a blue arrow points to the person's father. Pose the problem of determining the relation of each person to Kip.



The arrow picture determines the sex of most, but not all, of Kip's relatives. For example, there is not enough information to determine if Kip's grandchild and Kip's cousin are male or female.

Suggest that students draw their own family trees using the "you are my mother" and "you are my father" arrows.

NOTE: Only an aunt (uncle) who is a parent's sister (brother) can be included. The spouse of a parent's brother or sister is also an aunt or uncle, but they are not included in these family trees, which show only blood relations.

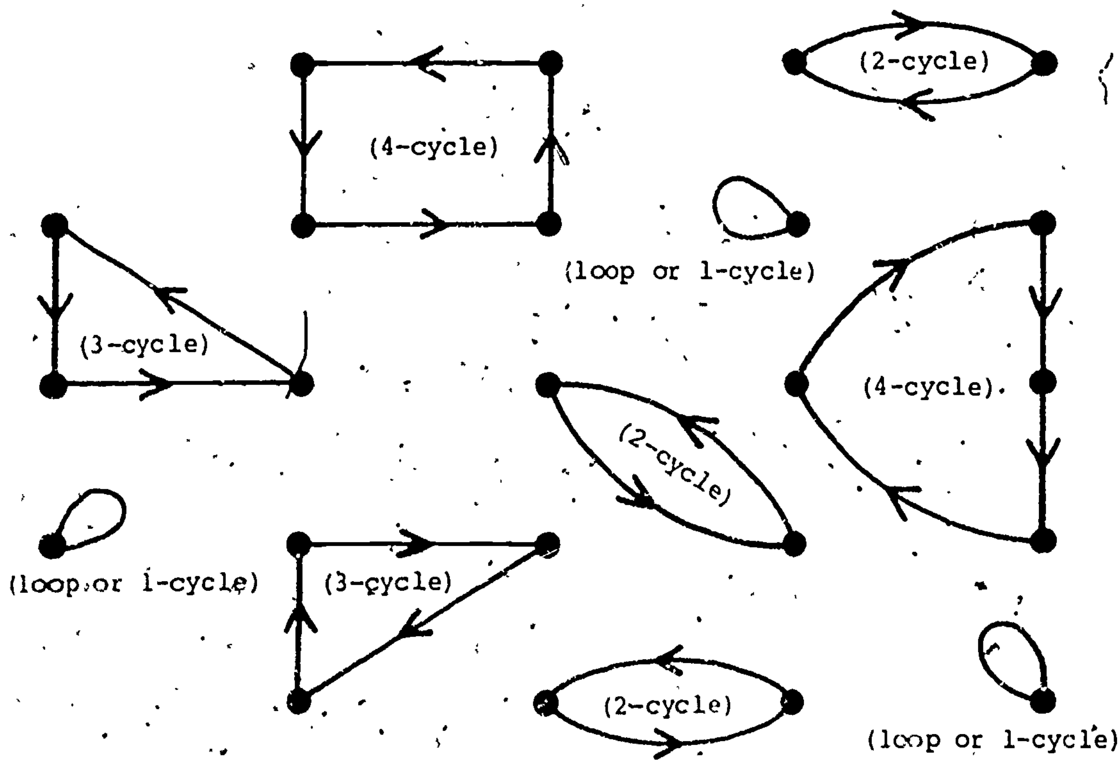
ACTIVITY B9: CHANGING SEATS

PREREQUISITE: None

OBJECTIVE: Given an arrow picture in which the dots represent seats in the classroom, students will change seats and make predictions about how many cycles it will take for all students to return to their original seats.

Exercise 1

Draw a picture of dots to represent the arrangement of seats for all students in the class. Help students to locate their seats in the picture. Do not label the dots. Draw arrows in your picture so that exactly one arrow starts at each dot and exactly one arrow ends at each dot. Your picture might look like the following. If your class is large enough, include at least one of each type of cycle.



Explain that this game is called "Changing Seats". Students should find the arrow that starts at the dot for their seat and determine whose seat they are going to move to. Before beginning the game, be sure everyone understands where to go. When everyone is ready, give the command, "Change".

Discuss which students are in new seats and which students are still in their own seats. (loops) Make a list of all students who are in their own seats after Round 1.

Change seats again. Discuss and list those students who are in their own seats after Round 2. Of course, students from Round 1 are included again.

Change again. Ask why the students on the first list are on every list.

Predict what will happen in Round 4 and make a list. Then change seats and correct the list if necessary.

Ask how many rounds of the game would have to be played before all students are back in their own seats. (12 rounds)

Exercise 2

On the board draw nine dots connected by red arrows as shown below. Explain that the dots represent the seats of players in "Changing Seats".

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Point to a dot and ask where that person there will go in Round 1. Repeat for some other dots. Point to a dot and ask where the person there will be after three rounds of the game. Repeat for some other dots. Ask how many rounds it will take for all of the children to return to their own seats. (15 rounds)

Label one of the dots in the 3-cycle b. Ask how many rounds it will take for that person to return. (0, 3, 6, 9, 12, ...) Encourage students to notice that these numbers are multiples of 3. Determine that the situation is the same for the other two dots in the cycle and label them a and c. Do the same for the other cycles, recording the information in a chart. Your chart should be similar to this one.

a,b,c	d	e,f,g,h,i
0	0	0
3	1	5
6	2	10
9	3	15
12	4	20
15	5	25
.	6	.
.	.	.
.	.	.

Ask again how many rounds it will take for all of the students to return to their own seats. Elicit the observation that the number of rounds must appear in all three columns. That is, the number must be a multiple of both 3 and of 5--that is, a multiple of 15 (0, 15, 30, 45, 60, ...).

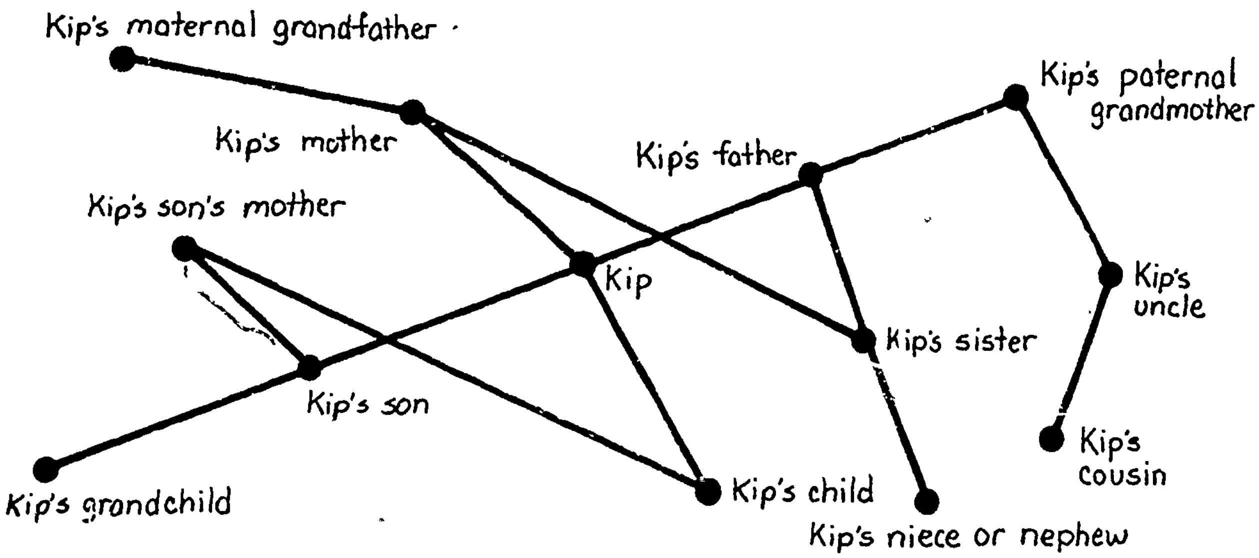
ACTIVITY B10: PARENTAL DISTANCE

PREREQUISITE: Activity B8

OBJECTIVE: Students will determine relationships through the use of cords representing parenthood.

Exercise 1

Draw and then review Kip's family tree from Activity B8 by labeling the dots. Then without erasing the dots or labels, replace all of the red and blue arrows by green cords.



Ask what the green cords could represent. ("you are my parent or child") Invite students to trace a road from Kip's son's mother to Kip's father, and a road from Kip's niece or nephew to Kip's mother. Explain that the parental distance between two people is the number of green cords in the shortest road connecting them in a family tree.

Let the students determine the parental distance between:

Kip and Kip's grandchild, (2)

Kip and Kip's uncle, (3)

Kip's son's mother and Kip's maternal grandfather, (4) and

Kip's mother and Kip's niece or nephew. (2)

Ask the students to name a pair of people who are a parental distance of 3 apart. Be sure that the shortest road is traced. There are many possible pairs including Kip and Kip's uncle or Kip's maternal grandfather and Kip's niece or nephew.

T: What is the largest parental distance in this picture? (6)
Which pairs of people are at a distance of 5?

Four answers are possible:

Kip's grandchild and Kip's uncle.

Kip's grandchild and Kip's niece or nephew.

Kip's son's mother and Kip's uncle.

Kip's maternal grandfather and Kip's uncle.

Exercise 2

Have each student draw their family tree and compute the distance from themselves to each relative in that tree. You may want to use one student's tree as an example for the rest of the class. After students complete their pictures, ask some questions.

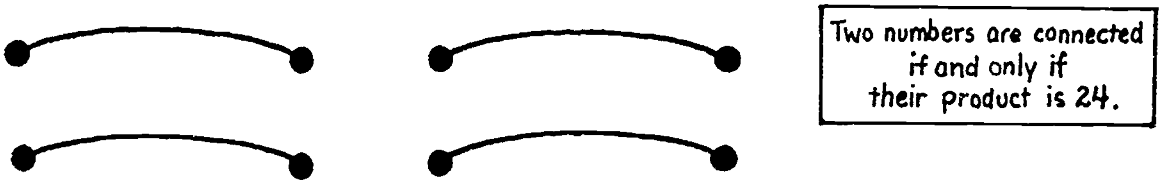
T: Can you find someone in your picture at a parental distance of 2 from yourself? (Sisters, brothers, and grandparents)
Is there anyone in your family tree who is at a parental distance of 3 from you? (Have some students draw the appropriate part of their family trees on the board to show the answer.)

ACTIVITY B11: MULTIPLICATION

PREREQUISITE: None

OBJECTIVE: Given a multiplication relation, students will identify pairs of ~~numbers~~ joined by cords.

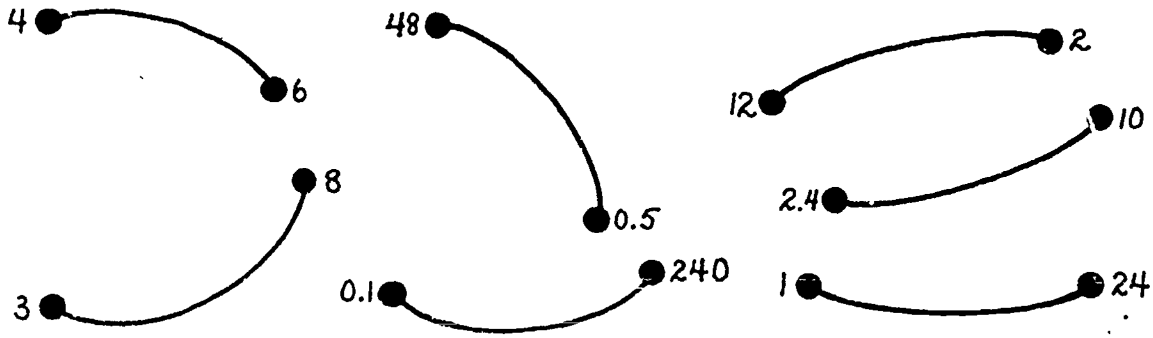
Draw this picture on the board.



T: Two numbers are joined by a red cord if and only if their product equals 24. Label each dot with any number so long as you obey the rule.

Let the students work independently. Then solicit labels for your picture and extend it with more dots and cords.

In the sample solution shown here, we have included some numbers other than integers. This can be an opportunity to work with decimals or a chance to use the hand-calculator to explore solutions.



Evidently, this exercise can be repeated whenever appropriate with other products replacing 24.

ACTIVITY B12: "IS A DIVISOR OF"

PREREQUISITE: None

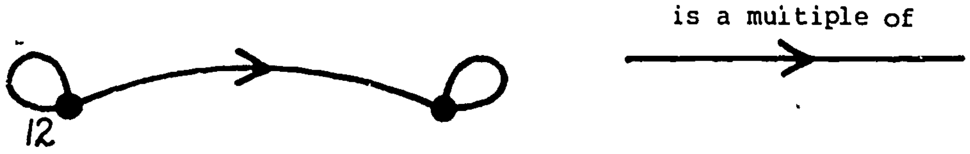
OBJECTIVE: Students will explore the relations "is a multiple of" and "is a divisor of".

Exercise 1

T: All dots are positive integers. The red arrow in this picture is for "is a multiple of". Which numbers could these dots be?

Ask students for several solutions. Each time, label the dots and trace the arrow as you read. For example, "10 is a multiple of 5".

Label the left dot "12".



T: What number could the other dot represent? (1, 2, 3, 4, or 6)

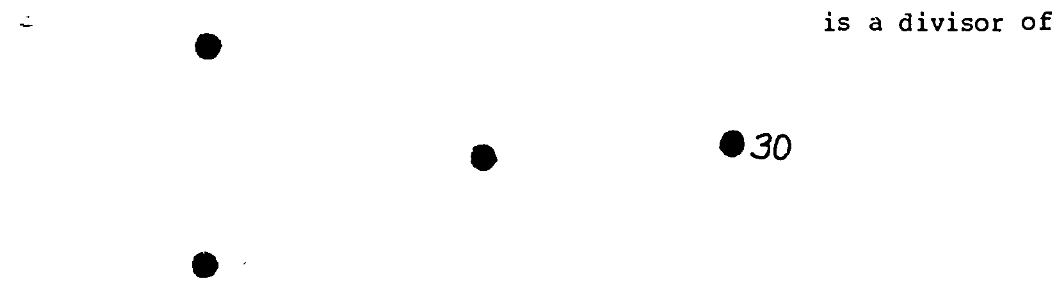
Augment your picture as follows.

T: The blue arrow is for "is a divisor of". It is the return arrow for the red arrow. For example, 12 is a multiple of 6, so 6 is a divisor of 12. And 12 is a multiple of 2, so 2 is a divisor of 12.

Replace 12 in the picture by 18. (The other dot could represent 1, 2, 3, 6, or 9.) Then repeat the exercise with 30. (1, 2, 3, 5, 6, 10, or 15) Emphasize the two ways of reading the arrow picture each time.

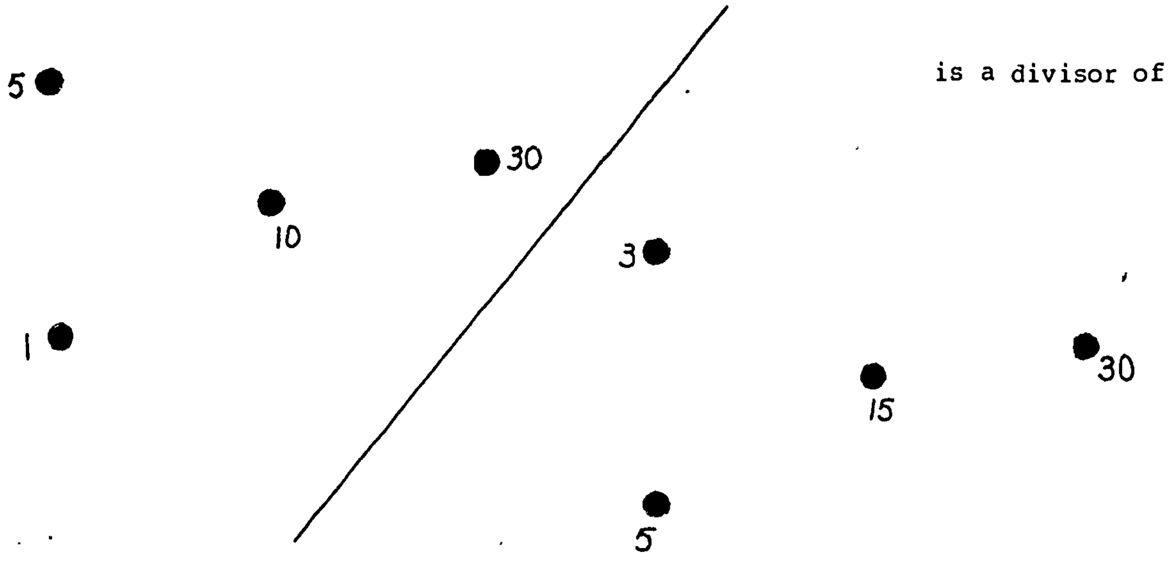
Exercise 2

Draw this picture on the board.



T: Copy this picture and label the dots. Then draw any missing blue arrows and loops.

Here are two of the many possible solutions.



ACTIVITY B13: A DETECTIVE STORY WITH ARROWS

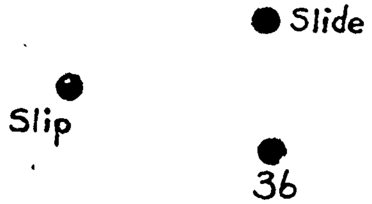
PREREQUISITE: Activity B12

OBJECTIVE: Students will solve a detective story with arrow pictures as clues.

First clue

Draw this picture on the board.

is a divisor of



T: Slip and Slide are two secret positive integers. What are some numbers they could be?

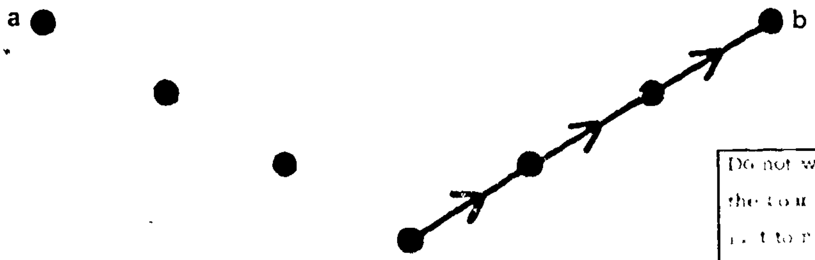
Record the students' conclusions on the board.

Slip could be 1, 2, 3, 4, 6, 9, 12, or 18.
Slide is more than Slip.

Second clue

T: Slip and Slide are the two largest numbers in this arrow picture.

+24



Do not write the letters on the board. They are here [illegible] the description of the [illegible] to follow.

By experimenting, students can determine that a and b are the two largest numbers and that b is 12 more than a. Then using the first clue you can construct the following table.

Slip	1	2	3	4	6	9	12	18
Slide	13	14	15	16	18	21	24	30

Third clue

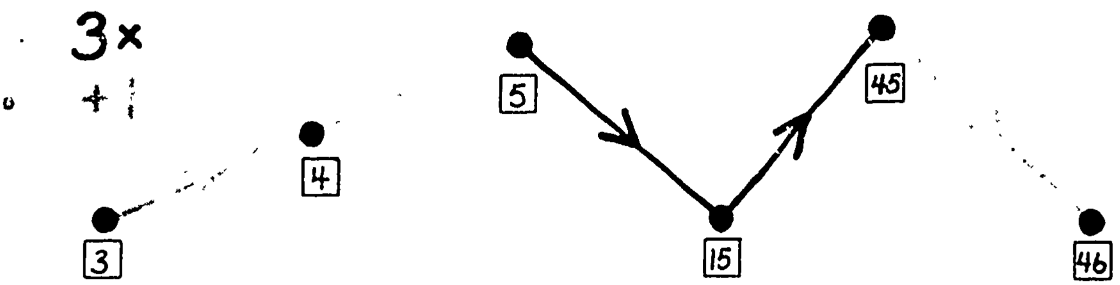
T: Slide has exactly four positive divisors.

Conclude that Slide could be 14, 15, or 21.

Fourth clue

T: Slip and Slide are on this arrow road.

Draw this picture on the board. The answers are in boxes.



The trick here is to try to fit in the arrow road the remaining possible pairs for Slip and Slide, namely: 2 with 14, 3 with 15, and 9 with 21. Only 3 and 15 can fit, so Slip is 3 and Slide is 15.

ACTIVITY B14: TALKATIVE NUMBERS

PREREQUISITE: Activity B3

OBJECTIVE: Students will construct roads with cords representing the rule: Two whole numbers are related if and only if one is a multiple of the other.

This lesson is very rich. It is intended to be extended over two or three class sessions as time and student motivation permit. At the beginning of each class session, review and practice the "Talkative Numbers" rule in a short warm-up.

Exercise 1

Distribute paper and red pencils. Begin by telling a story to your class. If necessary, remind the class that the whole numbers are 0, 1, 2, 3, 4, and so on.

T: For an entire month it rained continuously. In the Whole Numbers School, the numbers were not able to play outside, so they got noisier and noisier. Each day they thought of new tricks to play.

The number "0" is the principal of the school and "1" is the assistant principal. They met to figure out what to do to make the situation better and decided on a rule. In an assembly of the whole school, 0 announced a new rule: "From now on, two numbers may talk to each other if and only if one of them is a multiple of the other".

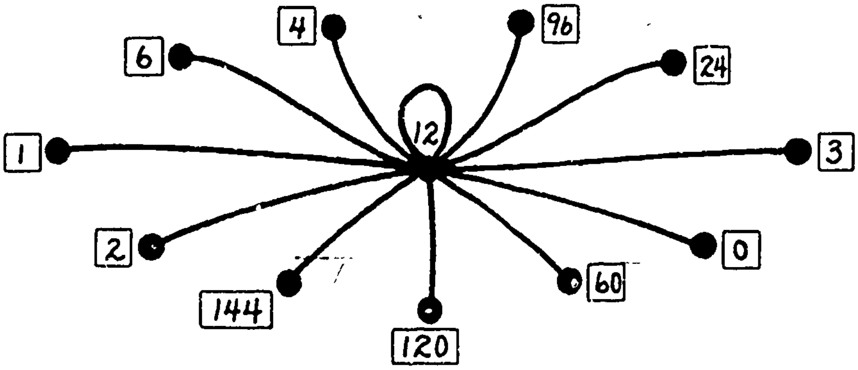
Write the rule on the board.

Two numbers may talk to each other
if and only if
one is a multiple of the other.

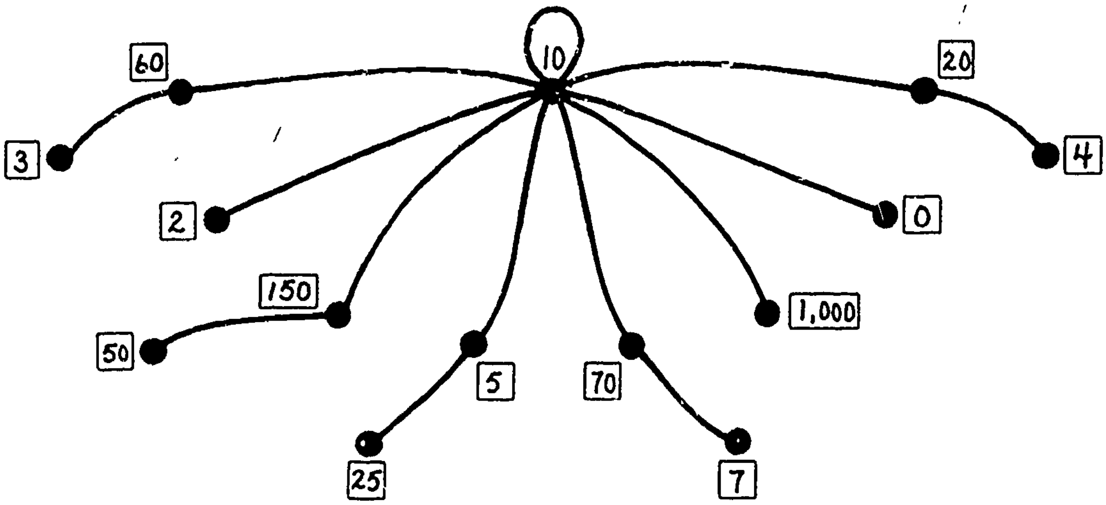
Ask the students to name pairs of numbers who can talk to each other.

T: To whom can 12 talk? We can show some of them in a picture. I will connect two numbers with a red cord if they may talk to each other.

Draw a picture similar to the following, and let students suggest labels for the dots. Check the arithmetic of each suggestion. Possible answers are in the boxes.

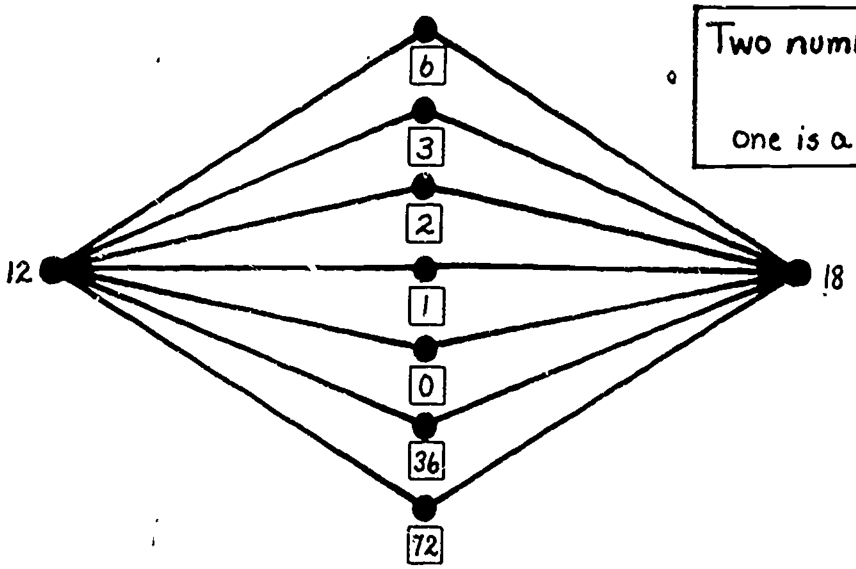


Then repeat the exercise with 10 and extend the picture to include two-step connections. Possible answers are in the boxes. Avoid labeling two dots with the same number.



Exercise 2

T: 12 and 18 are very good friends, but the rule does not allow them to talk to each other. How can 12 and 18 send messages to each other? Of course, they will find friends to help them. Can you name some of these friends?

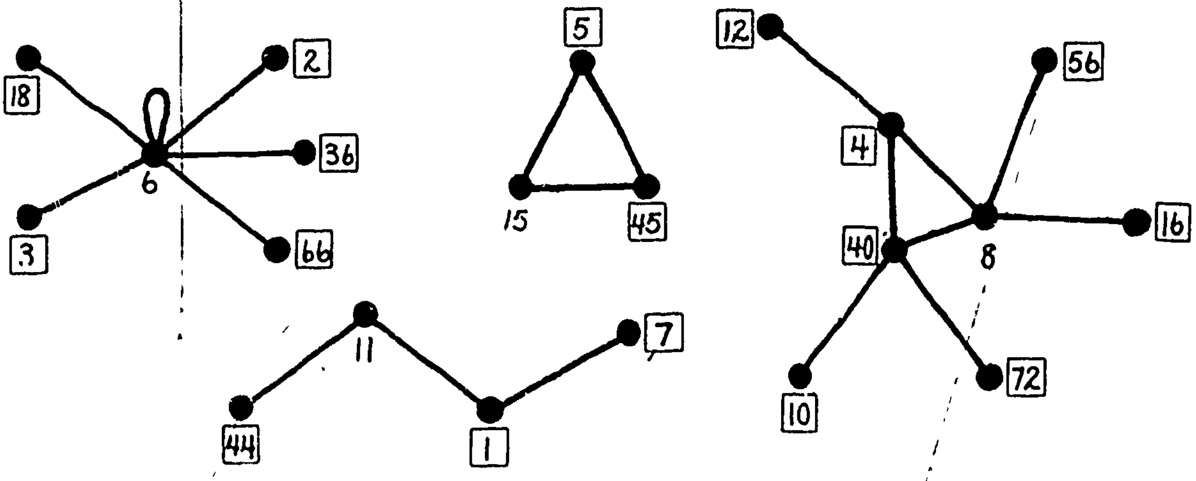


Two numbers talk to each other if and only if one is a multiple of the other.

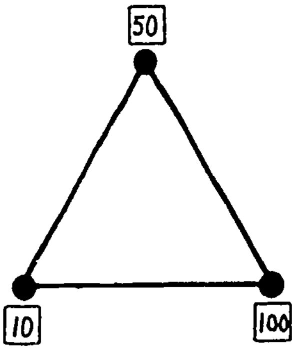
Note that any number that can relay a message is either a common divisor or a common multiple of 12 and 18. This is a good context in which to work with these two ideas using any pairs of numbers you like. In fact, you can ask students to look for a friend to handle messages among more than two friends. For example, 4, 6, and 10 send messages through the common multiples 0, 60, 120, 180, and so on and through the common divisor 1. The fact that the principal, 0, and the assistant principal, 1, can talk to all of the numbers emphasizes the fact that 0 is a multiple of all numbers and that 1 is a divisor of all numbers.

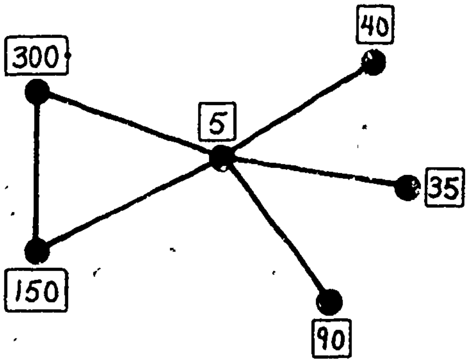
Exercise 3

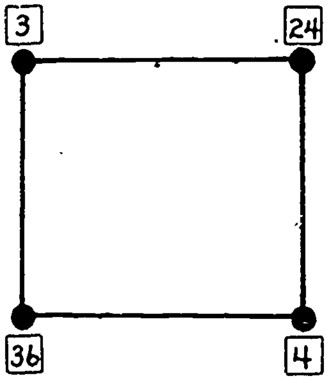
Many problems can be posed in this context. Use these pictures or devise others appropriate for your students' needs. Possible solutions are shown in the boxes.

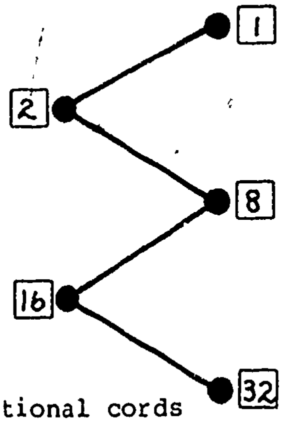


Another variety of problems is to propose cord pictures with no dots labeled. For example:

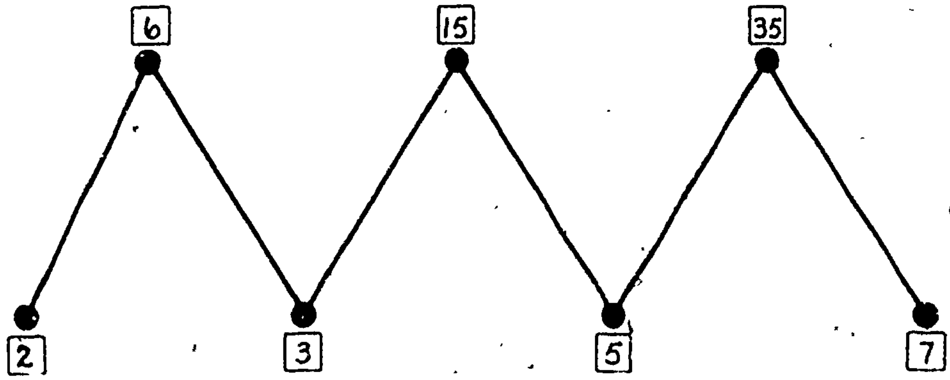








Additional cords can be drawn.



ACTIVITY B15: THE TELEPHONE GAME

PREREQUISITE: Activities B14 and N2

OBJECTIVE: Students will build cord roads, using the rule: Two integers are related if and only if one is the double of the other or one is 10 more than the other.

This activity is similar to Talkative Numbers in its richness. Divide the lessons into two or three class sessions at your discretion. Remember to review the Telephone Game rule and to do some warm-up exercises at the start of each session.

<u>Exercise 1</u>

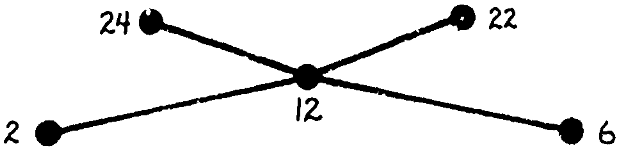
T: In the world of numbers the telephone system has a very strange wiring. There is a rule that tells whether two numbers can call each other.

Write the rule on the board.

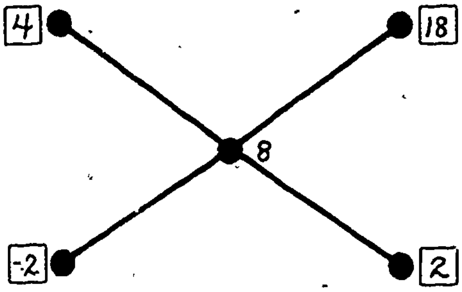
> Two integers can call each other by phone
> if and only if
> one of them is the double of the other
> or
> one of them is 10 more than the other.

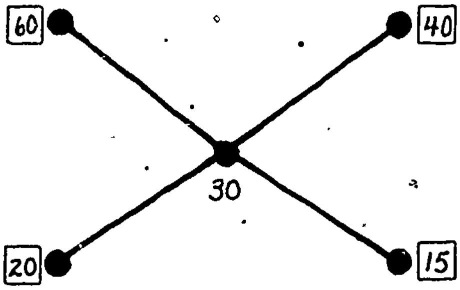
T: Whom can 12 call?

Let students apply the rule to find calling partners for 12. Check the arithmetic of each suggestion as it is made. Illustrate the results in a cord picture.

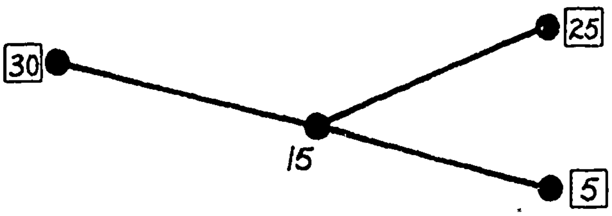


Ask the class to find the calling partners for a few other numbers, such as 8 and 30. You may need to remind the students that negative numbers are available and to help them find that 8 is 10 more than -2. Answers are in the boxes.





Next ask the class to find calling partners for 15. 15 can call 25, 5, and 30. Note that 15 is the double of 7.5, but that 7.5 is not an integer. If necessary, remind the class that you are restricting your game to integers. So 15 can only call three integer friends.

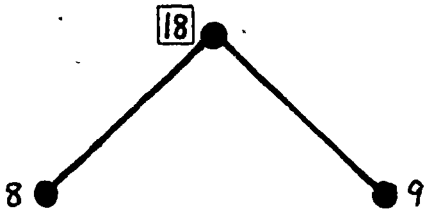


Ask the class to find the calling partners for other numbers of their choosing. When many have been found, ask if anyone has found another number that, like 15, can call only three integer friends. Some may suggest that all odd numbers can call exactly three friends and all even numbers can call exactly four friends. Should this happen, ask the students to find numbers that 0, 10, 20, -10, or -20 can call. 10, 20, -10, and -20 are the only even numbers who can call only three integer friends. 0 can only call 10 and -10.

Exercise 2

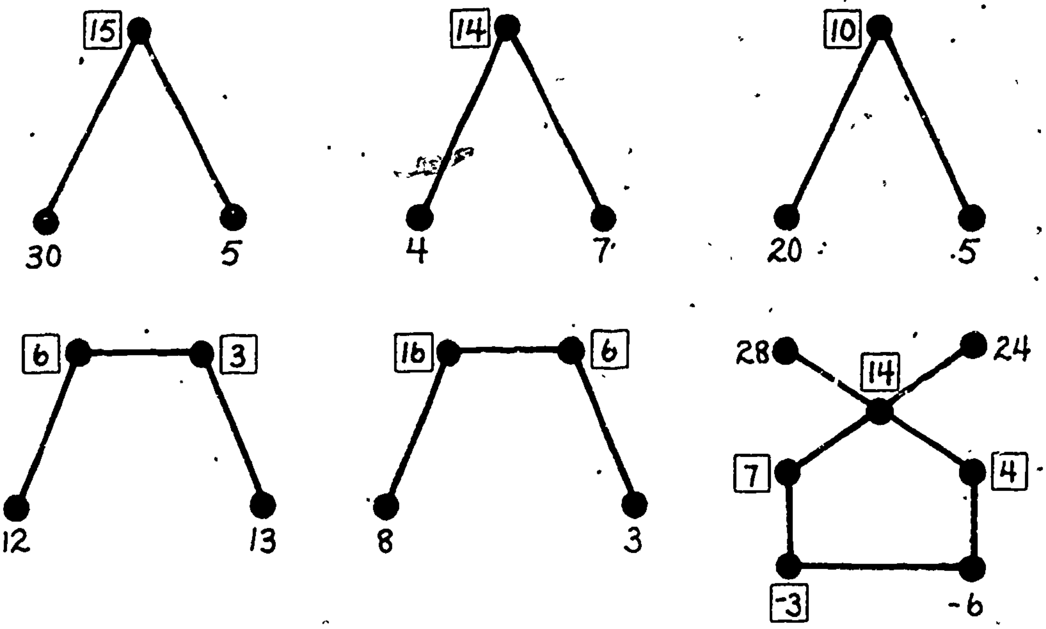
T: 8 cannot call 9. Can 8 send a message to 9 through a friend?

Draw a picture to illustrate the situation. The answer is in the box.



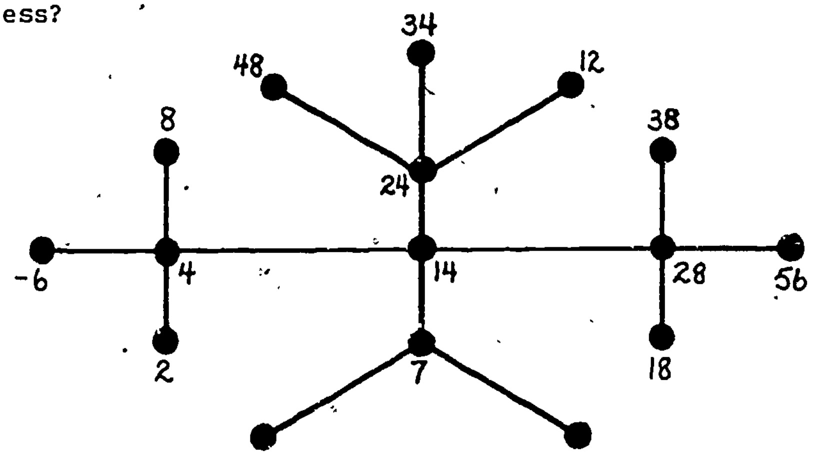
Two integers can call each other by phone
if and only if
one of them is the double of the other
or
one of them is 10 more than the other.

Here are a few similar problems. Answers are in the boxes.

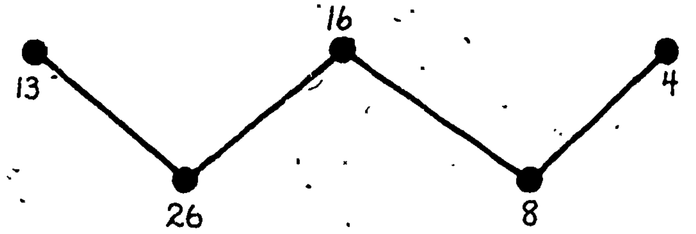


Exercise 3

T: It costs 10¢ to make a telephone call. Whom can 14 send a message to for 20¢ or less?



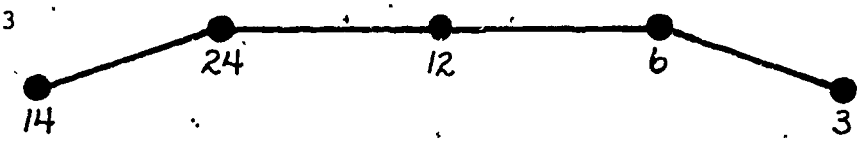
T: 13 cannot talk to 4 directly. At 10¢ for each telephone call, try to find the least expensive way for 13 to send a message to 4.



40¢

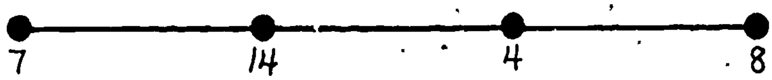
Here are several similar problems with possible solutions. They increase in difficulty.

1. 14 to 3



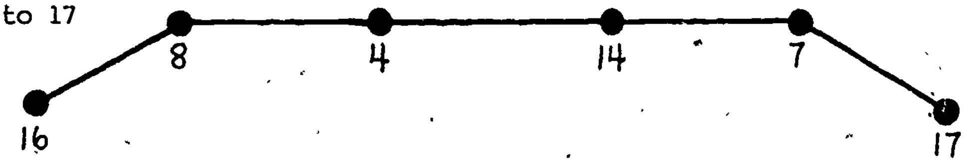
40¢

2. 7 to 8



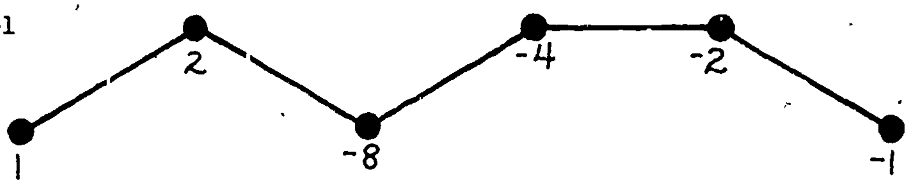
30¢

3. 16 to 17



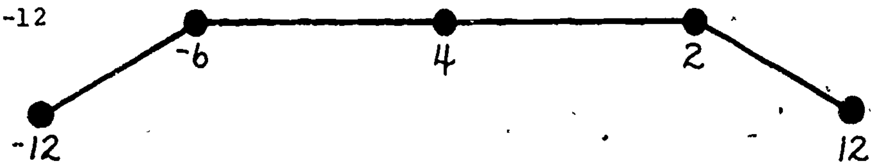
50¢

4. 1 to -1

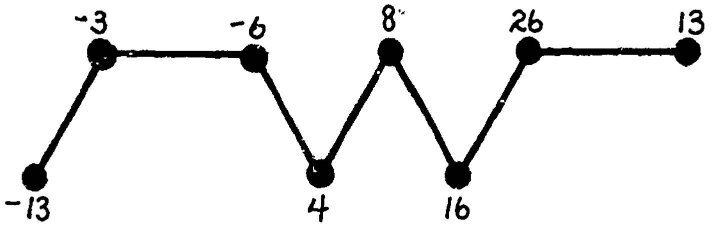


50¢

5. 12 to -12



40¢

6. -13 to 13 

70¢

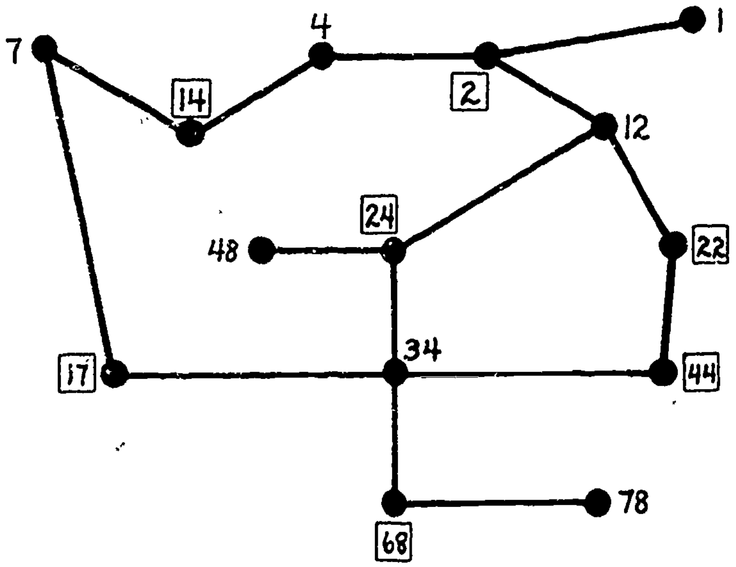
7. 15 to 16 (No solution)

Exercise 4

Lead the class in solving this detective story. Put cord pictures on the board as the story evolves. The answers are in boxes.

First clue

T: Zip is in this picture. Its dot is not labeled.

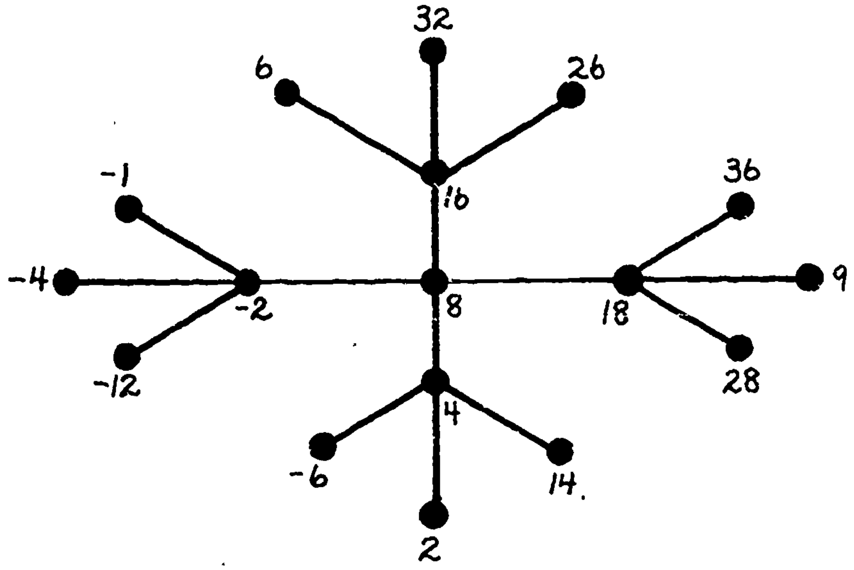


Second clue

T: Zip can send a message to 8 for exactly 20¢.

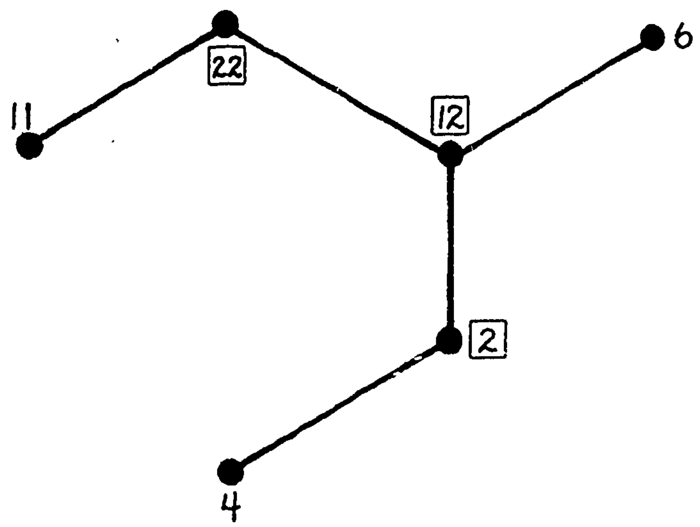
If necessary, remind the students that phone calls cost 10¢. Then let them work independently before recording their discoveries on the board. Comparing the results with those of the first clue yields 14 and 2 as possibilities for Zip.

ERIC Full Text Provided by ERIC



Third clue

T: Zip is in this picture, but its dot is not labeled. (Zip is 2.)



ACTIVITY B16: ONE-DIGIT DISTANCE

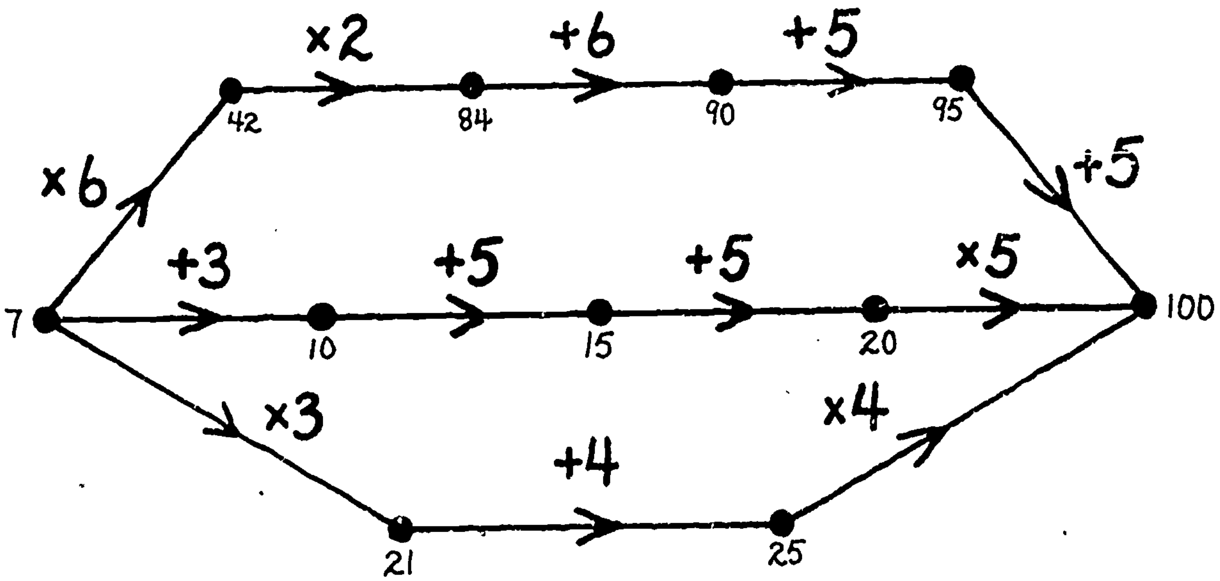
PREREQUISITE: Activity B3

OBJECTIVE: Students will build arrow roads with arrows representing operations by the whole numbers 1 through 9.

Exercise 1

T: Build a road from 7 to 100 using only arrows representing adding, subtracting, multiplying, or dividing by a whole number from 1 to 9. You may use more than one type of arrow. Only integers play in this game.

As with many of the arrow problems, there are many possible solutions. Three are shown here. Encourage the class to find several in order to make clear the variety of approaches that are possible.



T: There are many solutions to this problem, but let's look for a shortest road. The shortest we have found so far is one with three arrows. Can we find a shorter solution?

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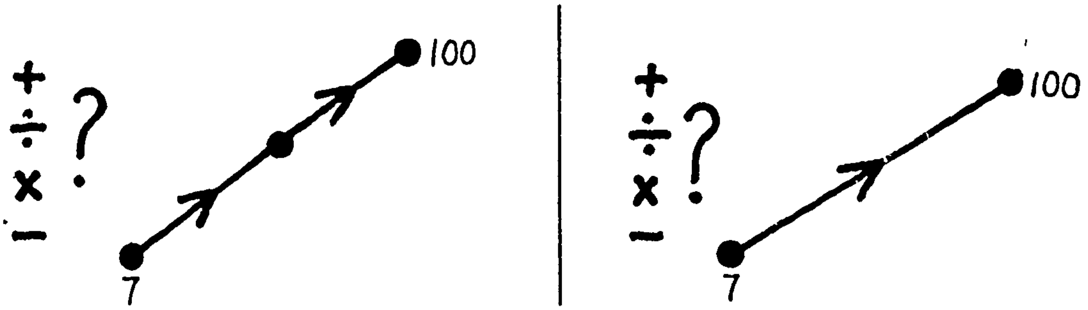
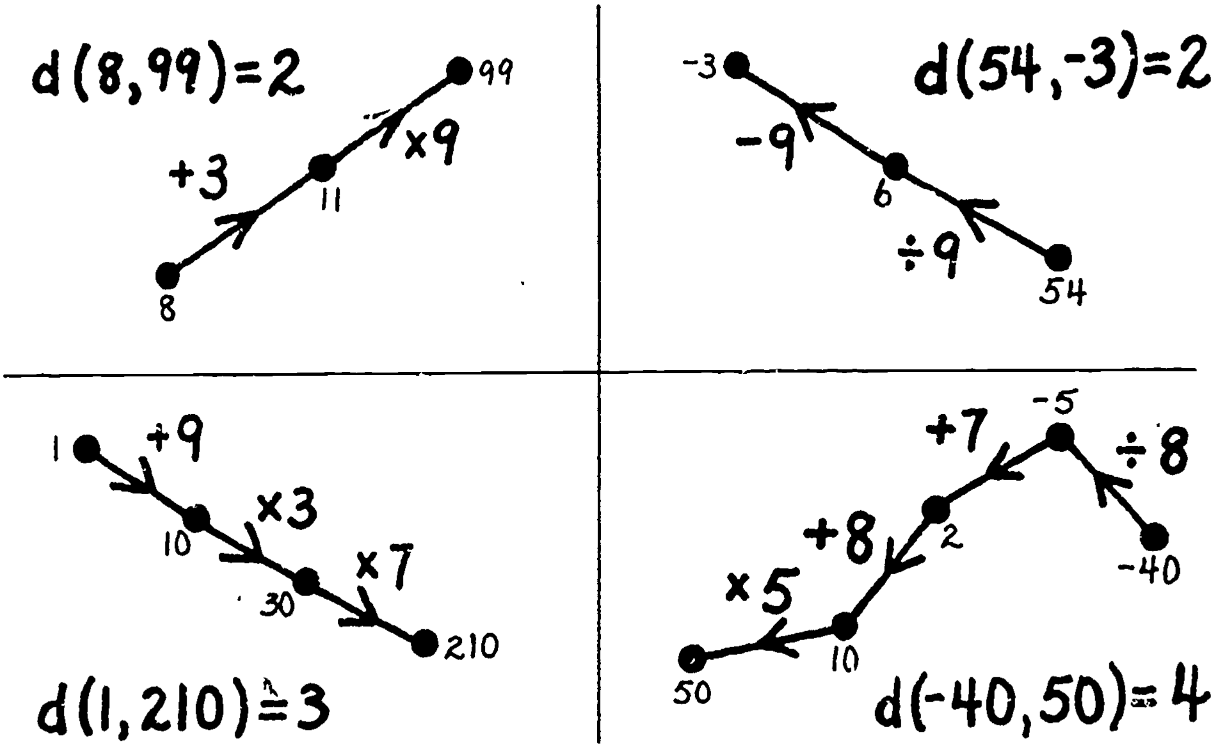
To facilitate the discussion, draw arrow pictures on the board representing one-arrow and two-arrow roads from 7 to 100. Then experiment with the possible labels for the arrows. The goal is to recognize that no successful labeling of the arrows that obeys the one-digit rule is possible.



T: The shortest road between 7 and 100 has three arrows, so we will say that the <u>one-digit distance</u> between 7 and 100 is 3. Write it this way:

$$d(7,100)=3$$

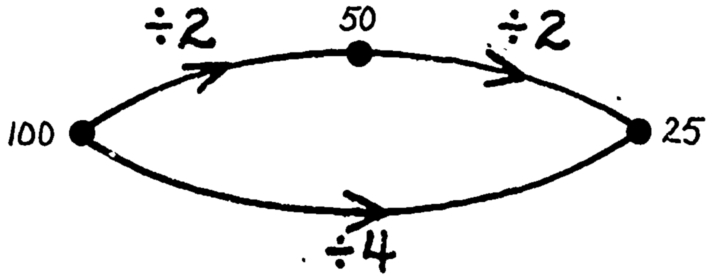
Have students determine one-digit distances by finding shortest roads between several pairs of numbers. For example:



Exercise 2

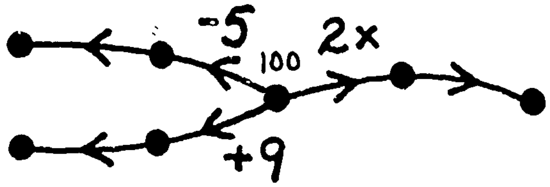
T: Let's find some numbers that are exactly two steps away from 100.

Illustrate on the board several examples that students suggest on the board. Be alert for roads that are not the shortest route between two numbers. For example:



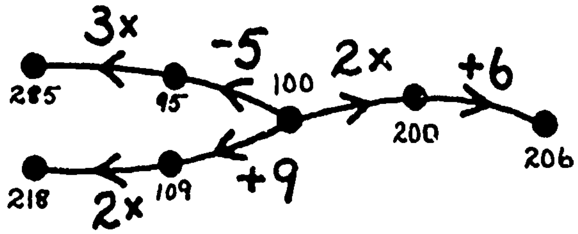
T: Let's find whole numbers between 200 and 300 that are two steps from 100.

Allow time for independent work and invite students to record some solutions on the board. If necessary, assist by drawing the following picture on the board.



T: Can we label the arrows so that the ending numbers are between 200 and 300?

One possible solution is the following:



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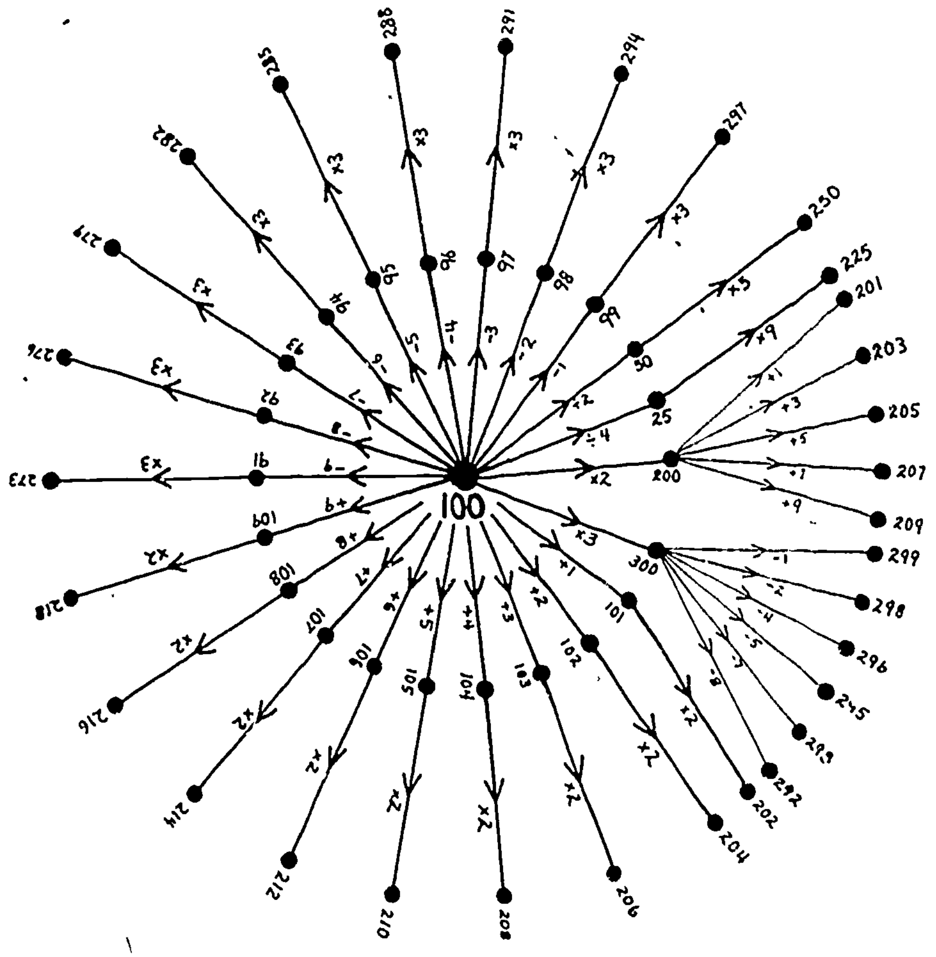
Let students continue to work independently. List numbers on the board as they are found. Finding a complete solution (one is shown below) might be a group project to be done outside of class.



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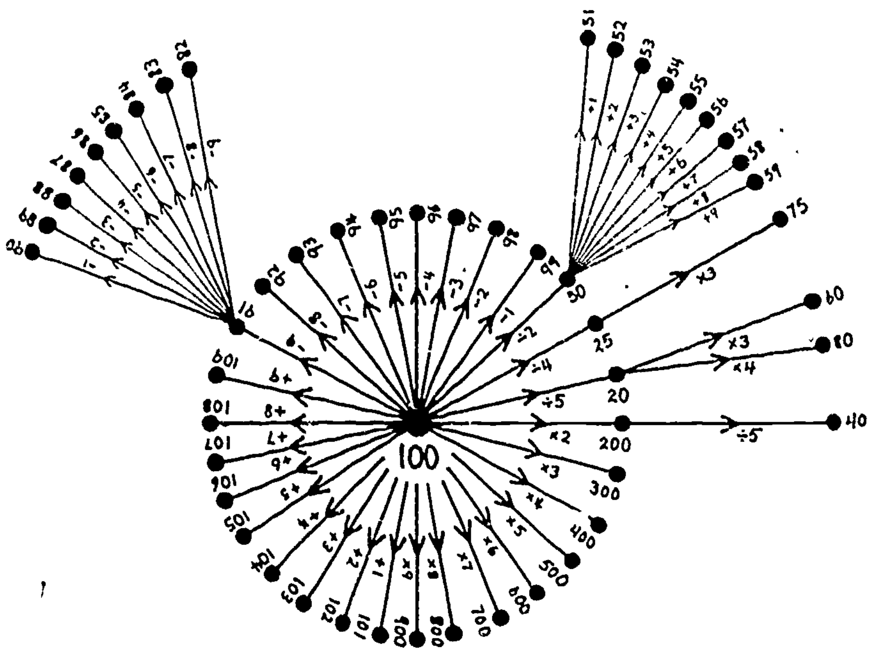
Exercise 3

Present a detective story about a secret number named "Tee".

First clue

T: Tee is a whole number between 50 and 100. The one-digit distance between Tee and 100 is 2. Which numbers could Tee be?

Let the students work independently on the problem for a few minutes. Assist students who have difficulty getting started. Discuss the clue collectively and help students develop a systematic method of finding the numbers by first thinking about numbers at a distance of 1 from 100. After finding the numbers at a one-digit distance of 1 from 100, the numbers at a distance of 2 from 100 (between 50 and 100) can be more readily determined. Conclude that Tee could be one of the numbers at the end of a two-arrow road starting at 100.



Second clue

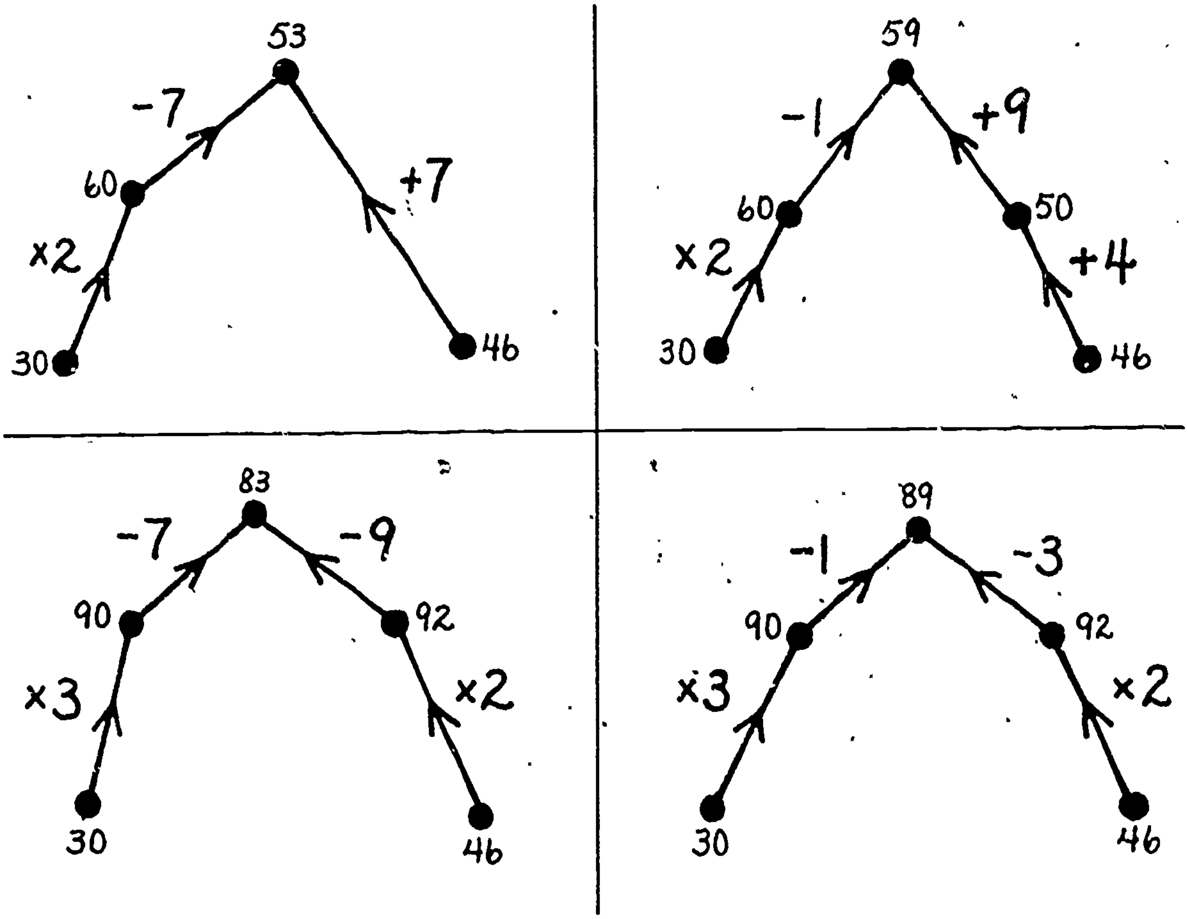
T: Tee is a prime number. Which numbers can we eliminate? Which numbers could Tee be?

Ask students to eliminate numbers that are not prime. Be sure to stop and discuss any number that is disputed. Conclude that Tee could be 53, 59, 83, or 89.

Third clue

$$d(\text{Tee}, 30) \neq d(\text{Tee}, 46)$$

Here are some roads students might draw to find Tee, which is 53.



ACTIVITY B17: DECIMAL DISTANCE

PREREQUISITE: Activities B3 and B15

OBJECTIVE: Using two types of cords, students will find the shortest cord road joining two whole numbers.

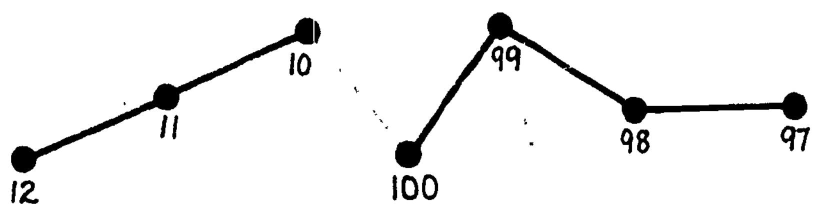
This lesson is rich in suggesting problems. It is intended to be taught in two or three class sessions, as time and student motivation permit. At the beginning of each class session, review and practice the cord rules in a short warm-up.

Exercise 1

Draw this picture on the board.

T: This game is like the Telephone Game, but we will be using two different types of cords to build roads between numbers. Red cords are for +1 or -1 and blue cords are for x10 and ÷10. Let's build a cord road between 12 and 97. Only whole numbers play in this game.

The following is the shortest route between 12 and 97; your students might suggest a longer route, in which case you should direct the discussion until the class discovers this route.



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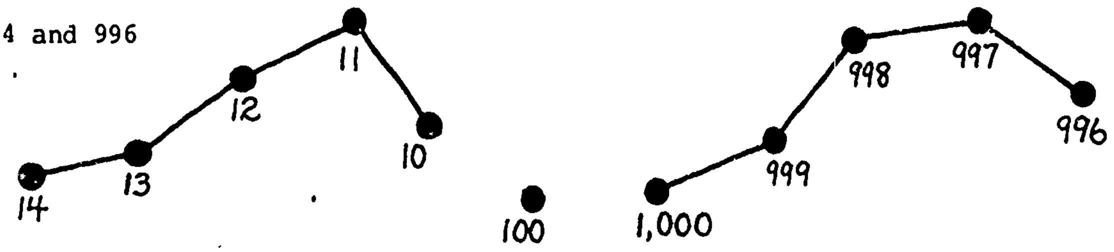
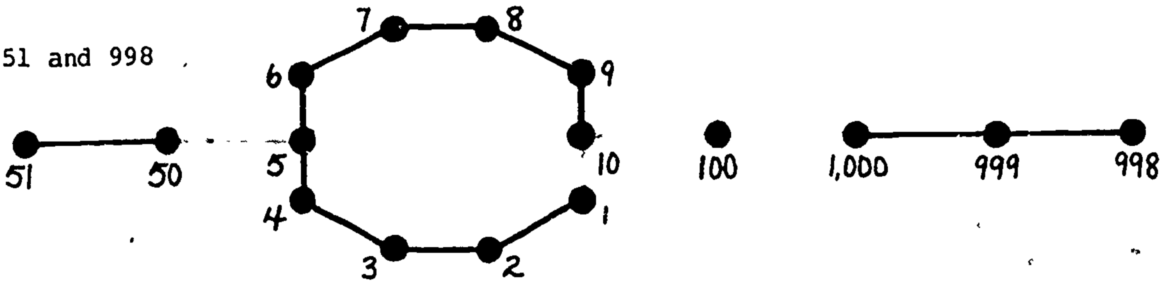
Repeat the activity to find a shortest road connecting 14 and 996 and connecting 51 and 998.

14 and 996

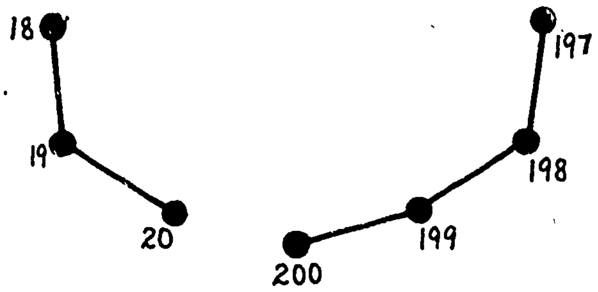


51 and 998

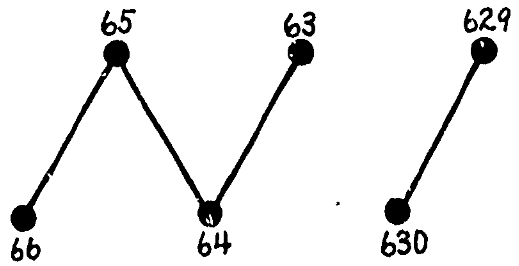


Ask students to work independently on the following roads (shown with solutions). Then have students draw their solutions on the board.

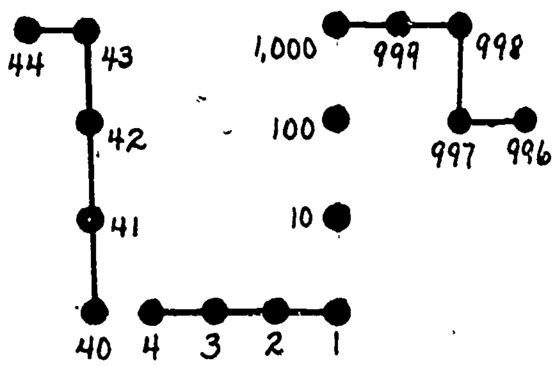
18 and 197



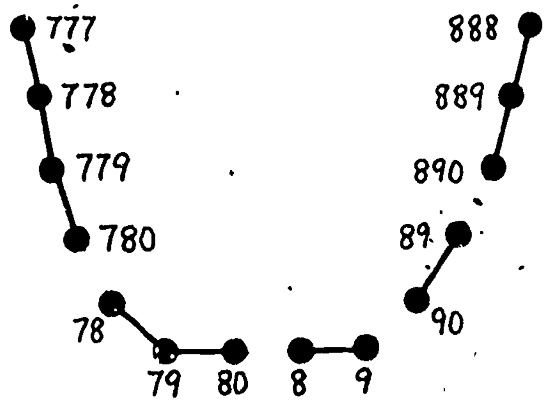
66 and 629



44 and 996



777 and 888



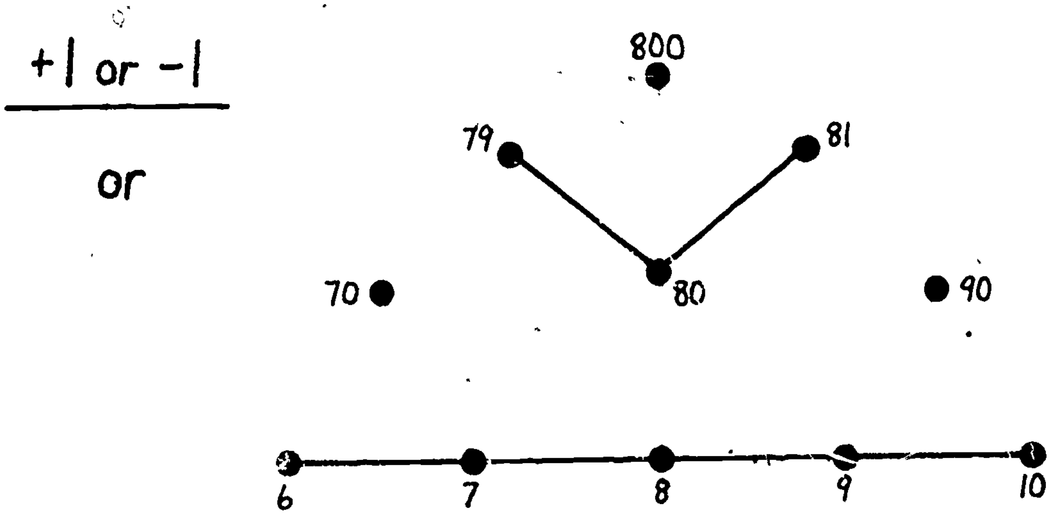
Exercise 2

T: This is a detective story about Bick.

First clue

T: The shortest cord road from Bick to 8 has two steps. What numbers could Bick be? (6, 10, 70, 79, 81, 90, or 800)

Build a cord picture as students suggest ending numbers. Your finished picture should look something like this.

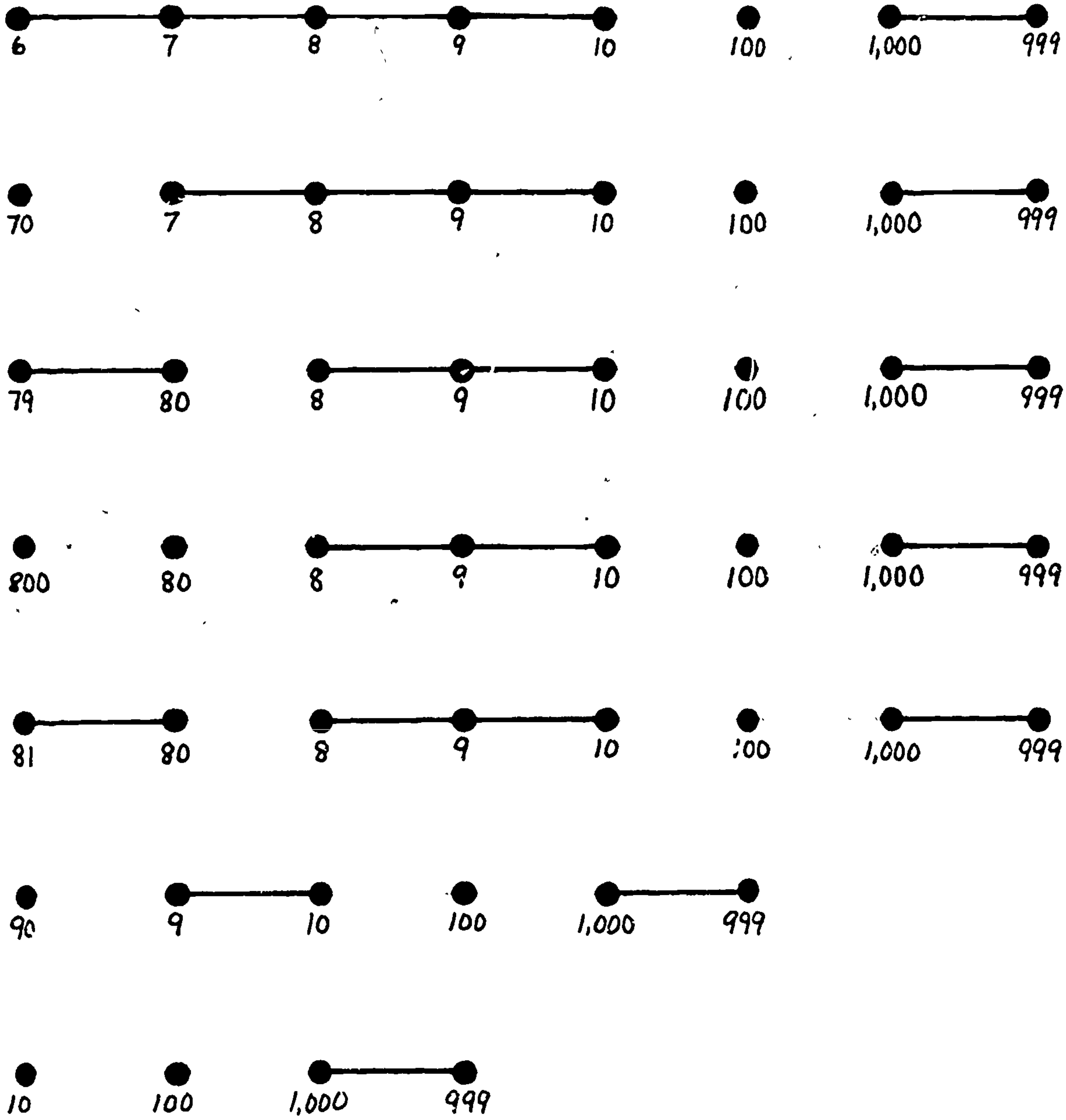


Second clue

T: The shortest cord road from Bick to 999 has three cords.

Lead a discussion of this clue with the students and decide that there are seven road problems to be tested (6 to 999, 70 to 999, 79 to 999, 800 to 999, 81 to 999, 90 to 999, and 10 to 999). Let students work independently or divide the work among seven small groups. Ask a representative from each group to present their road.

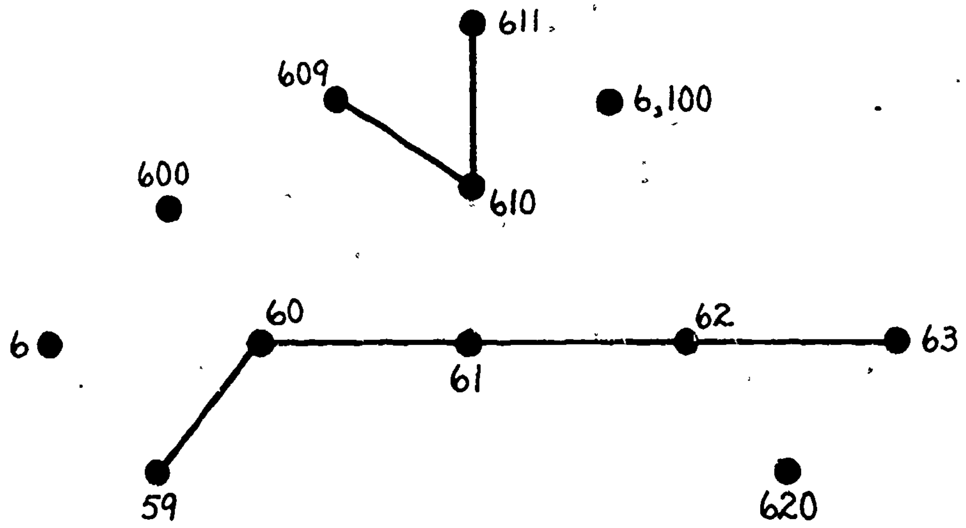
Shortest roads connecting the seven pairs of numbers are shown here. Evidently, Bick must be the number 10.



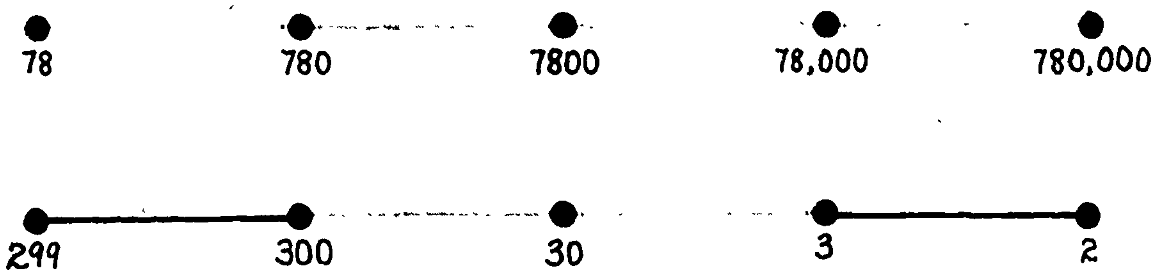
Exercise 3

Problems of this type can be freely chosen to meet the needs and interests of your class. Either choose two numbers and ask the class to find a shortest cord road connecting them, or choose a number and find all of the numbers a given number of steps away from it. Several more problems are shown below. These problems can evoke some systematic thinking about the construction of cord roads.

T: Find all the numbers that are exactly two steps from 61. (6, 59, 63, 600, 609, 611, 620, and 6,100)



T: What is the largest number that is exactly four steps from 78? (780,000)
What is the smallest number that is four steps from 299? (2)



ACTIVITY B18: PRIME FACTOR DISTANCE

PREREQUISITE: Activities B15, B17, and S10

OBJECTIVE: Student will find the prime factor distance between several pairs of numbers.

Exercise 1

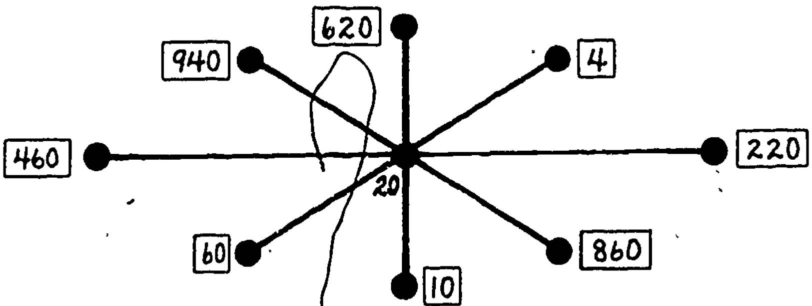
Have students list the positive prime numbers less than 50 and write them on the board. Remind them that prime numbers are those with exactly two divisors.

2	3	5	7	11
13	17	19	23	29
31	37	41	43	47

T: In the Telephone Game and the Talkative Numbers, we had rules for connecting two numbers. Now we will use another rule which uses prime numbers. I will write it on the board.

> Two whole numbers can be joined by a red cord
> if and only if
> one is a prime number times the other number.

Draw this cord picture on the board and invite students to suggest numbers for the dots. Possible answers are shown in boxes. For example, 20 = 5 x 4 and 5 is a prime number; 220 = 11 x 20 and 11 is a prime number; and so on.



Repeat the activity with several other numbers such as 12, 15, and 6.

<u>Exercise 2</u>

T: Let's find a shortest road connecting 10 and 16.

One of the many possible shortest roads is shown below.

Repeat the activity with several other pairs of numbers. Encourage students to draw more than one road for each problem. Several examples of shortest roads are shown below. Many solutions are possible.